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PRACTICAL GEOMETRY:

Or, A NEW and EASY

METHOD

OF

Treating that ART.

WHEREBY -

The PRACTICE of it is render'd plain and familiar, and the Student is directed in the most easy manner thro' the several Parts and Progressions of it.

Translated from the FRENCH of
Monsieur S. L E C L E R C.

THE FOURTH EDITION.

Illustrated with eighty COPPER-PLATES.

Wherein, besides the several Geometrical Figures, are contain'd many Examples of LANDSKIPS, Pieces of ARCHITECTURE, PERSPECTIVE, Draughts of FIGURES, RUINS, &c.

L O N D O N:

Printed for T. BOWLES, Print and Map-seller, in St. Paul's Church-yard; and J. BOWLES, Print and Map-seller, at the Black-Horse, Cornhill.

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John OF *Single his book*
GEOMETRY
IN
GENERAL.



Geometry is a Greek word, and in its native signification stands for no more than the measuring of land, but now we mean by it the principal part of Mathematics, which is a science that has continued quantity for its object.

That quantity is called continued quantity, which has all its parts conjoined; of this kind are all sorts of extension, magnitudes, and dimensions.

And these dimensions consist chiefly either in lines, or surfaces, or angles, or bodies, which last are not to be consider'd in respect of the quality of their matter, but of the extension of their parts.

Geometry is divided into speculative and practical.

The former is a science that teaches the mind how to form ideas of, and demonstrate the truth of geometrical propositions.

The latter, or practical Geometry, conducts the hand in working.

B

T H E

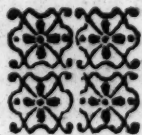


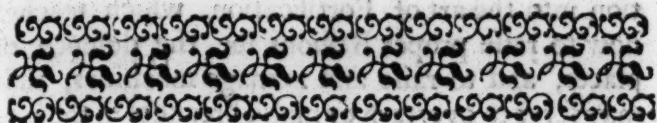
T H E
Original of Geometry.

Geometry had its original among the *Egyptians*, who were put under a necessity of inventing some such art to remedy the disorders, that commonly happened in their lands, by the overflowing of the *Nile*, which carried away their land-marks, and effaced the limits of their inheritances.

So that this practice, which in those days consisted only in measuring of land, that every one might have what belonged to him before the overflow, was called Geometry.

But in process of time, the *Egyptians* applied themselves to more subtle inquiries, and by degrees insensibly there arose from a practice altogether mechanical, a science that now holds the first place among all the others, according to its merit.





THE

Usefulness of Geometry.

Geometry is not only useful, but in several cases necessary. 'Tis owing to this, that Astronomers are put into a way of making their observations, coming at the knowledge of the extent of the heavens, duration of time, motions of the heavenly bodies, measures of seasons, of years, and of ages.

'Tis by the assistance of this science that Geographers present to our view at once, the magnitude of the whole earth, the vast extent of the seas, the divisions of Empires, Kingdoms, and Provinces

'Tis from this that the Architects take their just measures for the structure of public buildings, as well as of private houses.

By its help Engineers conduct all their works, take the situation and plan of places, measure their distances from one another, and carry their measure into places that are only accessible by the eye.

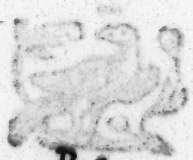
Persons of Quality, whose birth engages them to take the field, are obliged to apply themselves

to this science. It not only serves as an introduction into the art of Fortification, which teaches how to raise proper Bastions for the defence of places, and to raise and manage machines, that may serve to overturn or make breaches in those of the enemy; but also brings them to great skill and dexterity in the art of war, in forming an army in order for battle, in encamping, dividing the ground for quartering the army, taking maps of countries, plans of towns, forts, and castles, measuring all sorts of dimensions, both accessible and inaccessible, and in forming designs; finally to recommend them as much for their skill and address, as for their strength and courage.

All such as profess the Art of Designing, ought to know something of Geometry, seeing that without it they can't make themselves masters of Architecture, nor Perspective, which are two parts absolutely necessary to their art.



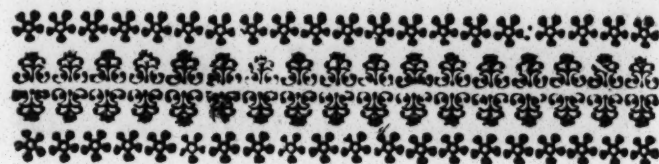
THE
 PRINCIPLES
 OF
 GEOMETRY.



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T H E
Principles of Geometry.

G EOMETRY is built upon three sorts of principles, *viz.* Definitions, Axioms, and Petitions.

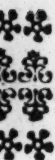
Definitions are brief explications of the names of things, or terms of art.

Axioms are propositions so true and evident, that 'tis impossible to question or contest their truth.

Petitions are demands so easy and intelligible, that the execution and putting them in practice, require no demonstration.



T H E



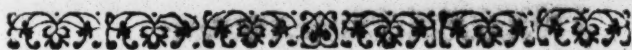
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H E

THE
DEFINITIONS.



THE

Definition of a Point.

A Point is that which has no Parts.

By this definition you may easily perceive, that a point has neither length, nor breadth, nor depth; that it is not any thing sensible, but only intellectual; for nothing falls under the notice of our senses, that has nothing of quantity; and nothing is quantity, that has not parts; so that to say a point is sensible, would be to say it has parts, which would contradict this definition. Notwithstanding since no operation can be perform'd without the intervention of something corporeal, we usually represent a mathematical point by a physical point, which is an object of sight the smallest and the least sensible that can be, and which has no geometrical magnitude divisible to our senses, and is made by the prick of a pin, point of a compass, pen or pencil, as the point marked

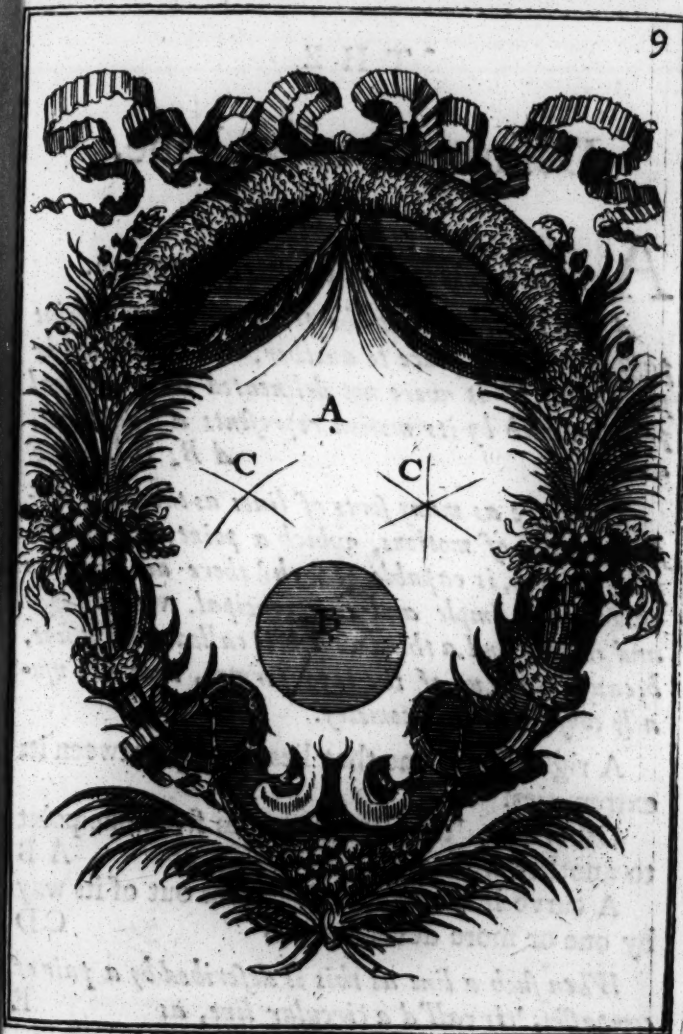
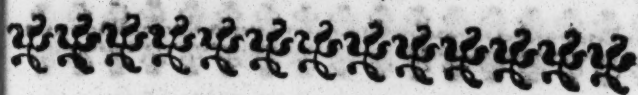
A

A central point, or centre, is a point from which a circle, or a circumference is described; or rather it is the middle of a figure, as the point

B

A secant point, or as some call it, a point of intersection, is a point where two or more lines cross one another, as the point

C





THE

Definition of a Line.

A Line is a length without any breadth.

A line is nothing but the track made by a point passing from one place to another, and would not be perceived, if it were not delineated by a physical point, which by its motion represents a line to us, as A B, C D, E F

There are as many sorts of lines as there are different kinds of motions, which a point, the principle of a line, is capable of; tho' there are but two which are simple and the principal, viz. a right and curve, and a third which is called a mixt line, because made up of the two former, that are usually considered in Geometry.

A right line is one that lies equally between its extremities.

Otherwise, 'tis a line that goes from one point to another without any deviation, as A B

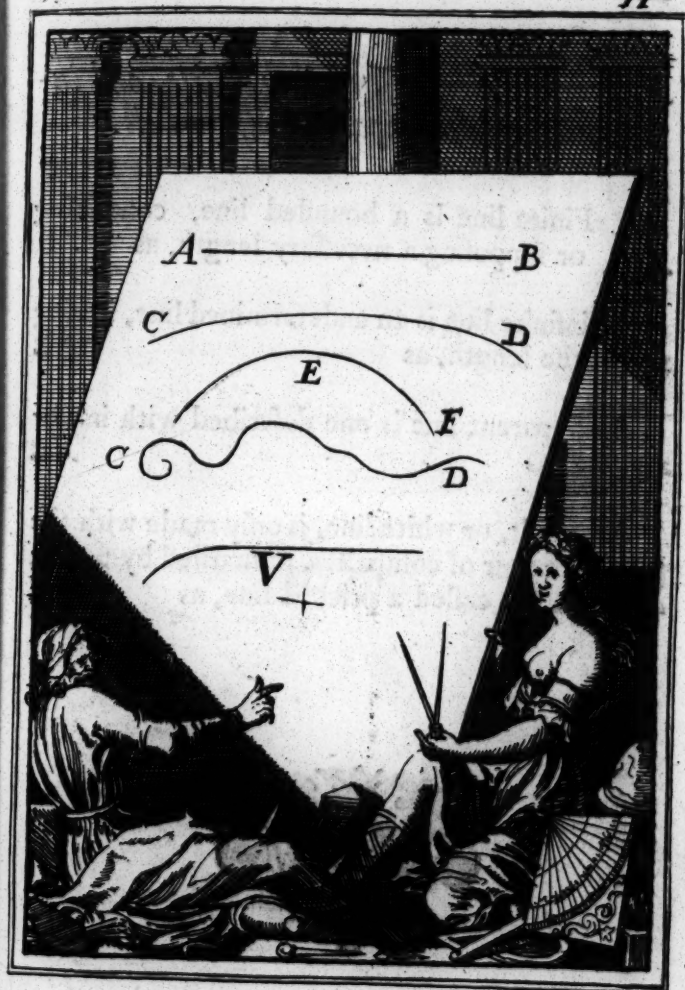
A curve line is that which turns out of its way by one or more deviations, as C D

When such a line as this is described by a pair of compasses, 'tis call'd a circular line, as E

A mixt line is that which is both straight and a curve, as the line V



II





A line is distinguished into finite and infinite, into apparent and occult.

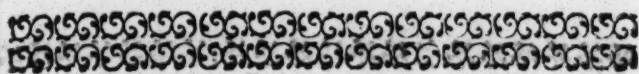
A Finite line is a bounded line, containing or supposing a necessary length, as **A**

An infinite line is an undetermined line, having no precise length, as **B**

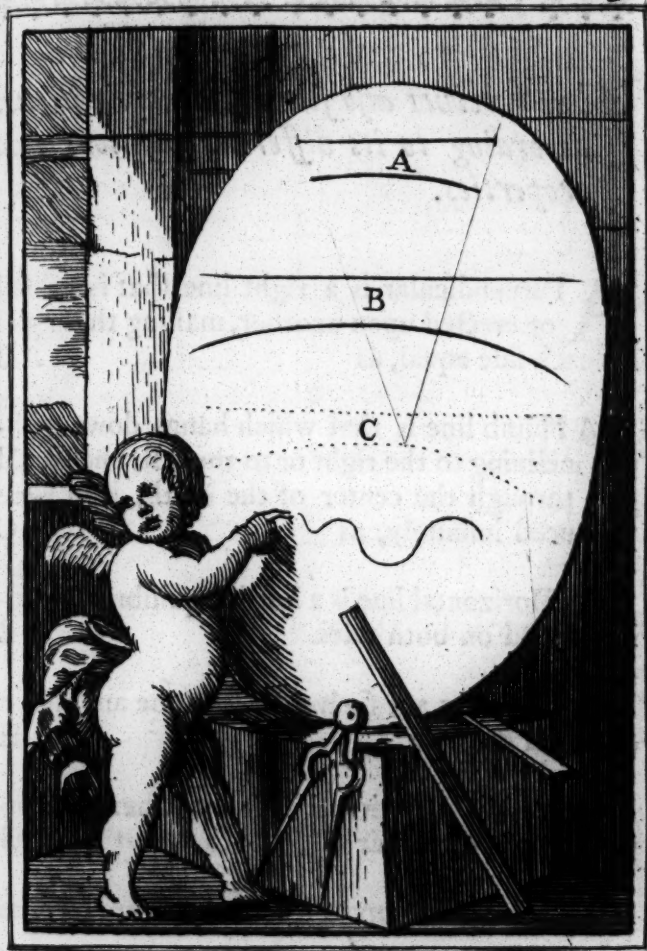
An apparent line is one described with ink or a pencil, as **A B**

An occult, or white line, is only made with the point of a pair of compasses, or marked by points, and then 'tis called a prick'd line, as **C**





13





A line receives also several denominations, according to its different positions and properties.

A Perpendicular is a right line that is let fall or erected upon another, making the angles on each side equal, as A B

A Plumb line is that which hangs down without inclining to the right or to the left, and would pass through the center of the earth, if it were produced infinitely, as C

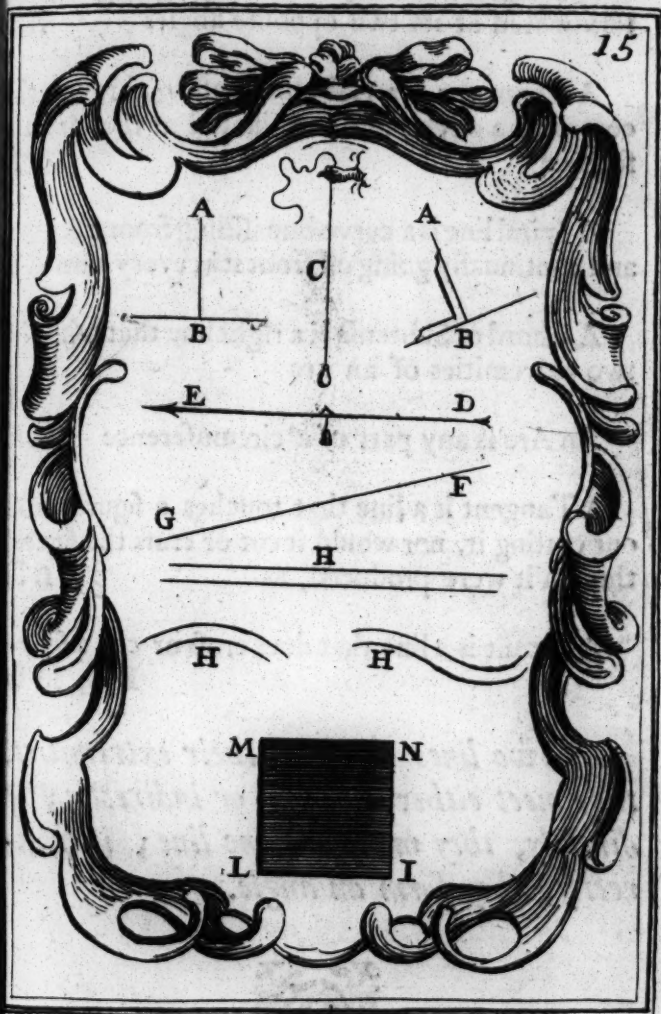
An Horizontal line is a line in equilibrio, equally inclin'd on both sides. D E

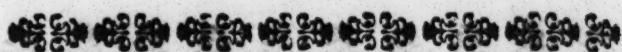
Parallel lines are such as follow one another at an equal distance H

An Oblique line is one that is neither horizontal nor perpendicular F G

A Base is a line upon which the figure rests, as I L

Sides are the lines that contain a figure, as I N, L M





A Diagonal is a right line crossing a figure, and terminated at its two opposite angles A B

A Diameter is a right line passing through the centre of a circle, and terminated at the circumference C D

A Spiral line is a curve line issuing from a centre, and continually going off from it at every turn E F

A Chord or Subtense is a right line that joins the two extremities of an arc G H

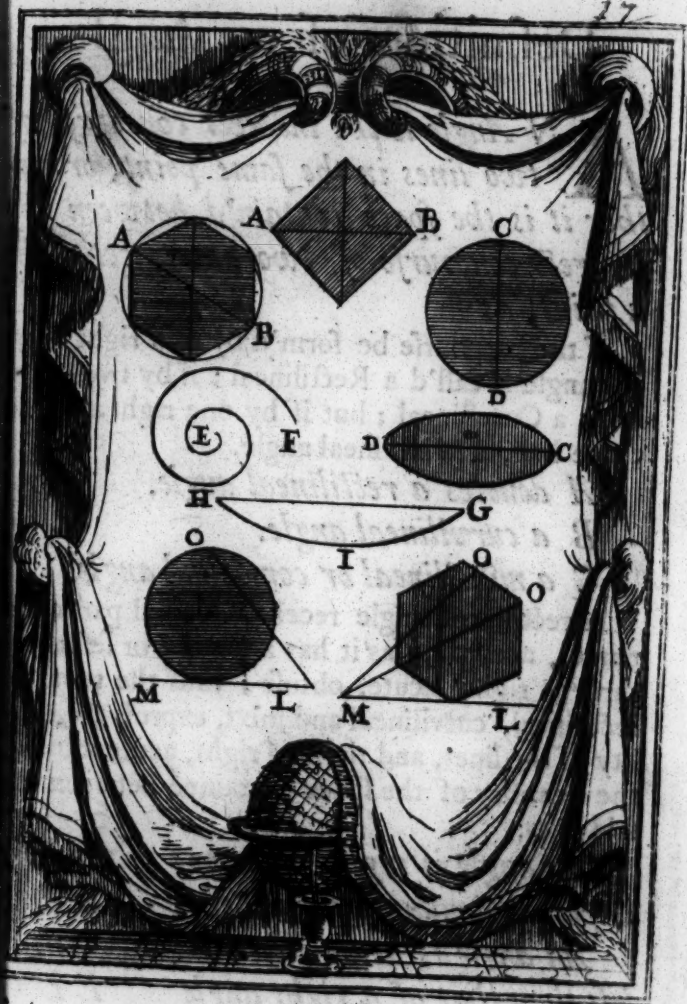
An Arc is any part of a circumference G I H

A Tangent is a line that touches a figure without cutting it, nor would it cut or cross the figure, though it were produced, as L M

A Secant is a line that does cross or cut a figure L O, M O

If two lines meet at their extremities, they meet either directly or indirectly; if directly, they make but one line; if indirectly, they form an angle.







T H E
Definition of an Angle.

AN *Angle is the indiret concurrence of two lines in the same point, or rather it is the space contain'd between the indiret concurrence of two lines meeting in a point, as*

If the concurrence be form'd by two right lines, the angle is call'd a Rectilineal; if by two curve lines, a Curvilineal; but if by one right and one curve line, a Mixtilineal angle.

A denotes a rectilineal angle.

B a curvilineal angle.

C a mixtilineal or compound angle.

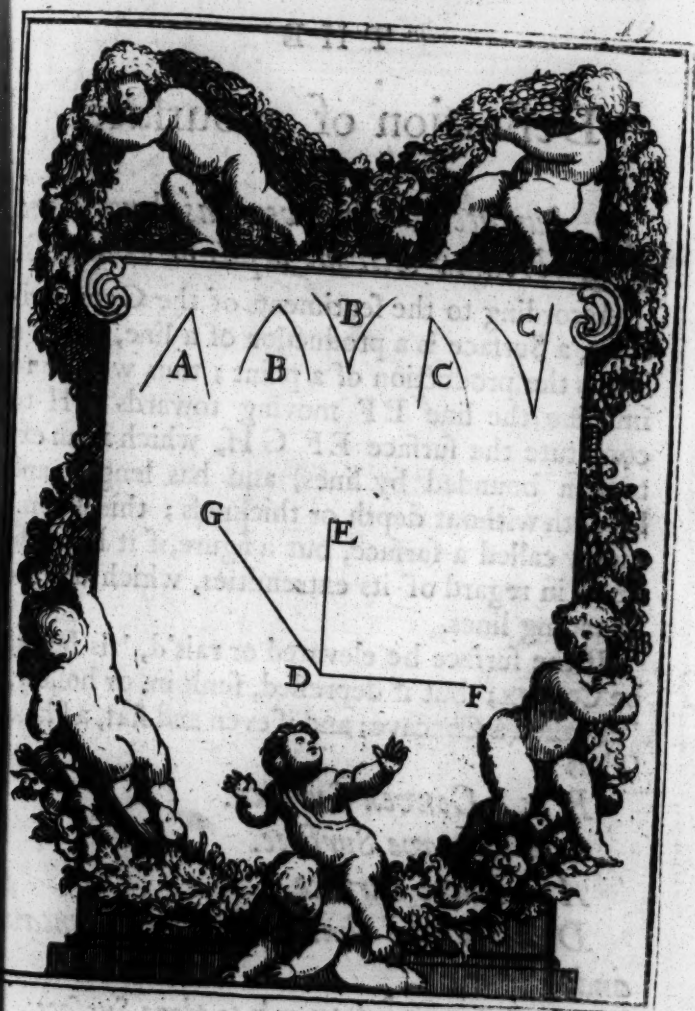
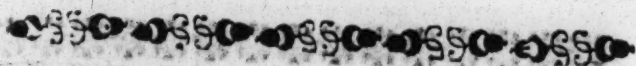
A rectilineal angle receives several particular names, according as it has a greater or less aperture, as right, acute, obtuse; thus the terms of rectilineal, curvilineal and mixt, express the quality of the lines, and those of right, acute, obtuse, the quantity of the space contained between the said lines.

An angle is right, when one of the lines is perpendicular to the other E D F

An angle is acute when its aperture is less than that of a right angle E D G

An angle is obtuse when its aperture is greater than that of a right angle F D G

The middle letter D denotes the angle.





THE

Definition of a Surface.

A *Surface is whatever has length and breadth without depth or thickness.*

According to the sentiments of the Geometricians, a Surface is a production of a line, just as a line is the production of a point; thus we are to imagine the line EF moving towards GH to constitute the surface EF GH, which is an extension bounded by lines, and has length and breadth without depth or thickness; this is commonly called a surface, but a figure, if it be consider'd in regard of its extremities, which are the bounding lines.

If the surface be elevated or rais'd, 'tis said to be Convex; but if depressed, sunk in, or hollow, 'tis called a Concave; and if even and flat, a Plane. Thus

B is a Convex Surface.

C a Concave Surface.

A a Plane Surface.

D a Surface that is Convex, Concave and Plane.

This first part relates only to plane Surfaces.

The terminus, term or boundary of any thing is its extremity: thus a point is the terminus of a line, a line is the term of a surface, and a surface is the terminus of a body.



Of Surfaces or Figures that are
Rectilineal.

*Surfaces take their particular names from
the number of their sides ; thus*

A **I** S a Trigon or triangle, a figure with three sides.

B a Tetragon or square, a figure of four sides.

C a Pentagon, or a figure of five sides

D an Hexagon, a figure of six sides.

E an Heptagon, or figure of seven sides.

F an Octagon, or figure of eight sides.

G an Enneagon, or figure of nine sides.

H a Decagon, or figure of ten sides.

I an Hendecagon, or figure of eleven sides.

L a Dodecagon, or figure of twelve sides.

*All these figures are also call'd by the
general name of Polygons.*

OF TRIANGLES.

*Triangles are distinguished by the quality of
their angles, and by the disposition of their
sides. Thus*

M is a right-angled Triangle, *i. e.* has one right angle.

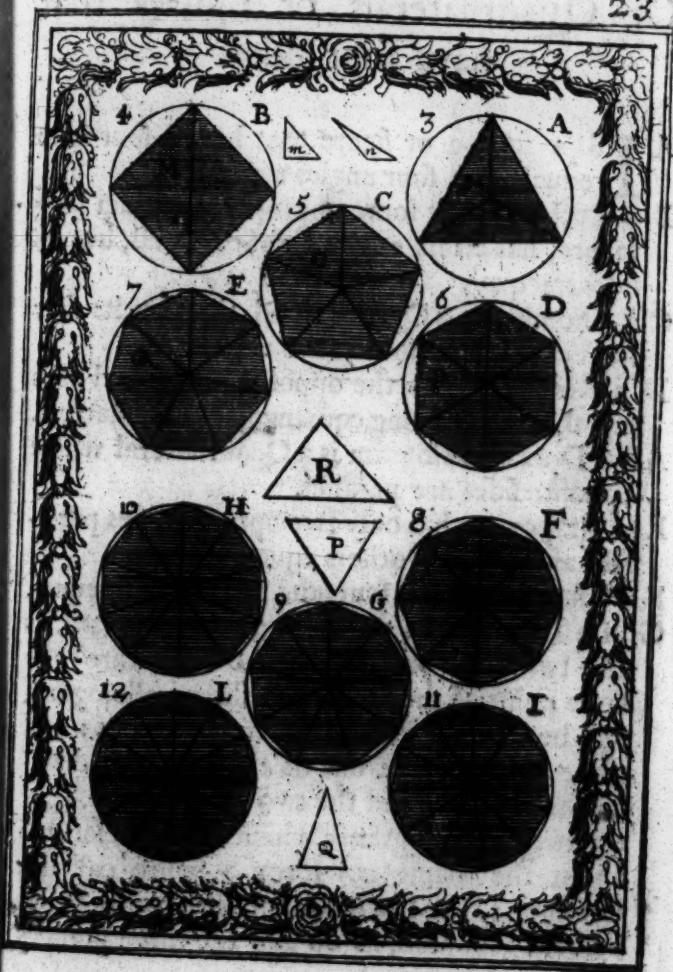
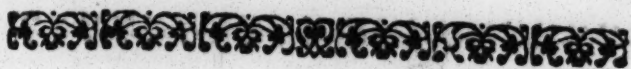
N an obtuse-angled Triangle, *i. e.* has one obtuse angle.

O an acute-angled Triangle, *i. e.* has all three angles acute.

P an equilateral Triangle, *i. e.* has its three sides equal.

Q an Isosceles Triangle, *i. e.* has only two sides equal.

R a Scalene Triangle, *i. e.* has all its three sides unequal.

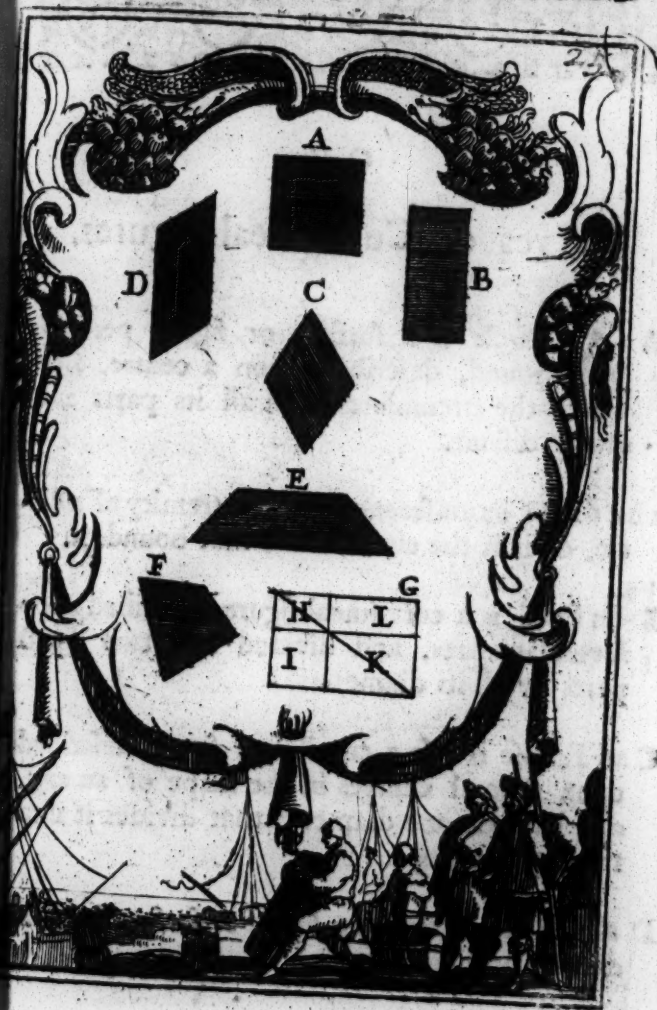




Of Quadrilaterals or figures that have four sides.

- A **I**S a square or figure that has its four sides equal, and four angles right.
- B a Rectangle, by some improperly call'd a long square, has all its angles right or equal, but its sides unequal.
- C a Rhombus is a Quadrilateral that has its four sides equal, but not its four angles.
- D a Rhomboid has the opposite angles and sides equal, without being equiangular or equilateral.
- A B C D a Parallelogram is a Quadrilateral whose opposite sides are parallel.
- E a Trapezium has only two opposite sides parallel, and the two others equal.
- F a Trapezoid has its four sides and angles unequal.
- G if a Diagonal be drawn in a parallelogram, as also two lines parallel to the sides, thro' the same point of the Diagonal, the parallelogram will be divided into four parallelograms; and three of them, viz. one of those described upon the diameter and the two supplements, (*i. e.* the two Parallelograms, which are not described about the diameter,) form a figure called a Gnomon; thus the three parallelograms H I L make a Gnomon, as do also the three parallelograms I K L

All figures having more than four sides, are call'd Polygons or Multilaterals.





O F

Curves or Curvilinear figures.

A Circle is a surface or figure perfectly round, described upon a centre, from which the circumference in all its parts is equally distant.

a b c d. A Circumference is the extremity of a circle, or it is the circular line that bounds it.

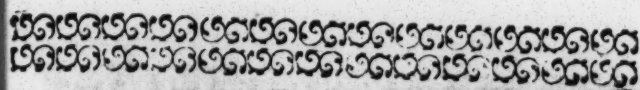
B an Oval is a curvilinear figure described upon several centres, and divided into two equal parts by all its diameters.

C an Ellipse is also a curvilinear figure described upon several centres in the shape of an egg, and has but one diameter that divides it into two equal parts.

D a Volute or Scroll is a figure or surface bounded by a spiral line.

E is a Cylindric surface.

F is an irregular curvilinear figure, compos'd of several dissimilar curve lines.



THESE FIGURES ARE TO BE USED IN THE FOLLOWING PROPOSITIONS



O F

Compound Figures.

A Semicircle is a figure contained between half the circumference and the diameter.

B a portion of a circle is a figure comprehended within any part of a circle and a right line.

g a large portion of a circle is greater than half the circle

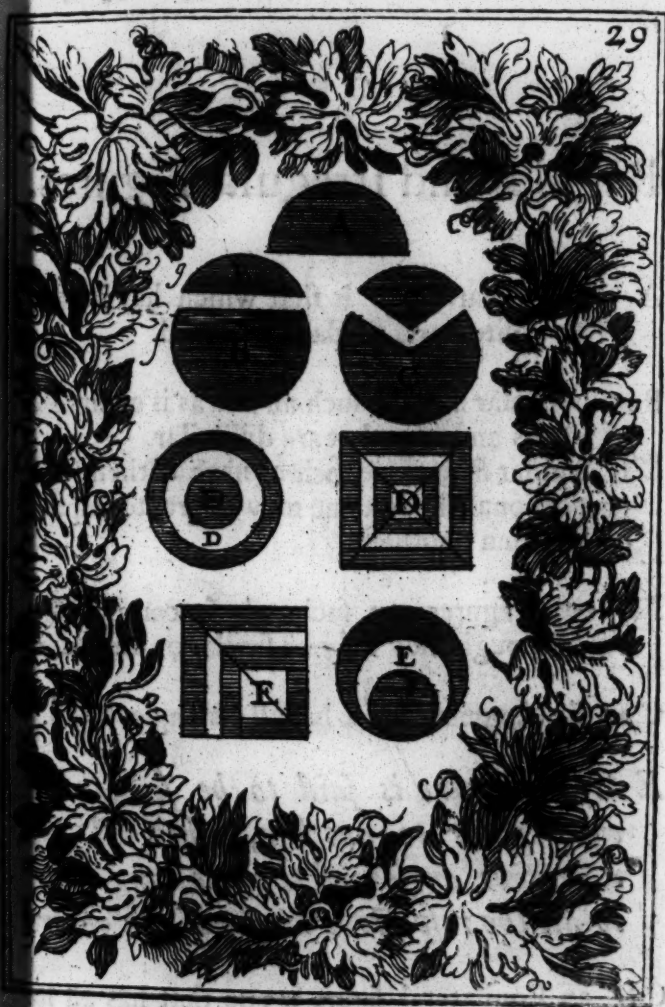
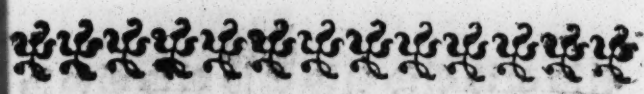
f a small portion of a circle is that which is less than half the circle.

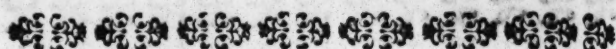
C a Sector is a figure contained between two semi-diameters and an arc, greater or less than a semicircle.

There is also a large or small Sector.

D Concentric figures are such as have the same centre.

E Excentric figures are such as are described upon different centres.





O F

Regular and Irregular Figures.

A Regular figure is that which has its opposite parts similar and equal.

B an Irregular figure is such an one as is compos'd of angles and sides that are dissimilar.

E E Similar figures are such as have all their sides proportional, though one may be greater, equal, or less than another.

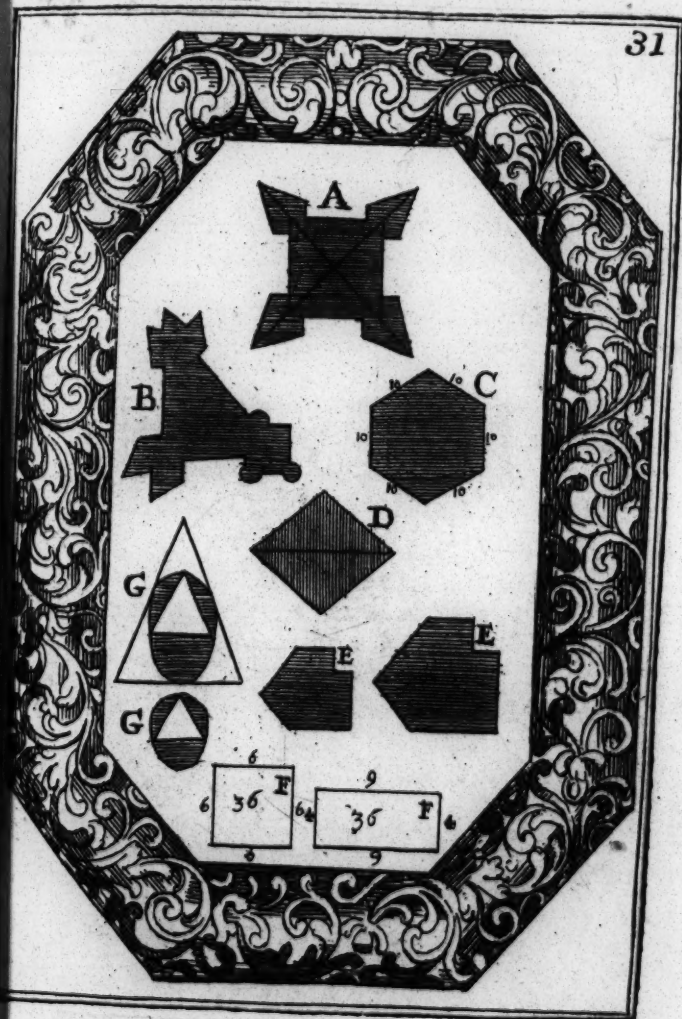
F F equal figures are such whose contents are equal, though they may be similar or dissimilar.

C an Equiangular figure has all its angles equal.

E E one figure is said to be similar or equiangular to another, when all the respective angles of the one, are equal to all the respective angles of the other.

C D an Equilateral figure is one that has all its sides equal.

G G Similar curvilinear figures are such as will admit similar Polygons to be inscribed in them, or circumscribed about them.



A

THE
AXIOMS.

D



A X I O M S.

I.

Things equal to the same third, are equal to one another.

The lines A C, A C, which are equal to A B, are also equal to one another.

II.

If to equal things, equal things be added, the whole will be equal.

The lines A C, A C are equal,

The lines C D, C D added are equal

The wholes A D, A D are also equal,

III.

If from equal things, equal things be taken away, the remainders will be equal.

If from the equal lines

you take away the equal lines

the remaining parts

will be also equal

A D, A D

A C, A C

C D, C D

IV.

If to unequal things, you add equal things, the whole will be unequal.

If to the unequal lines

you add the equal lines

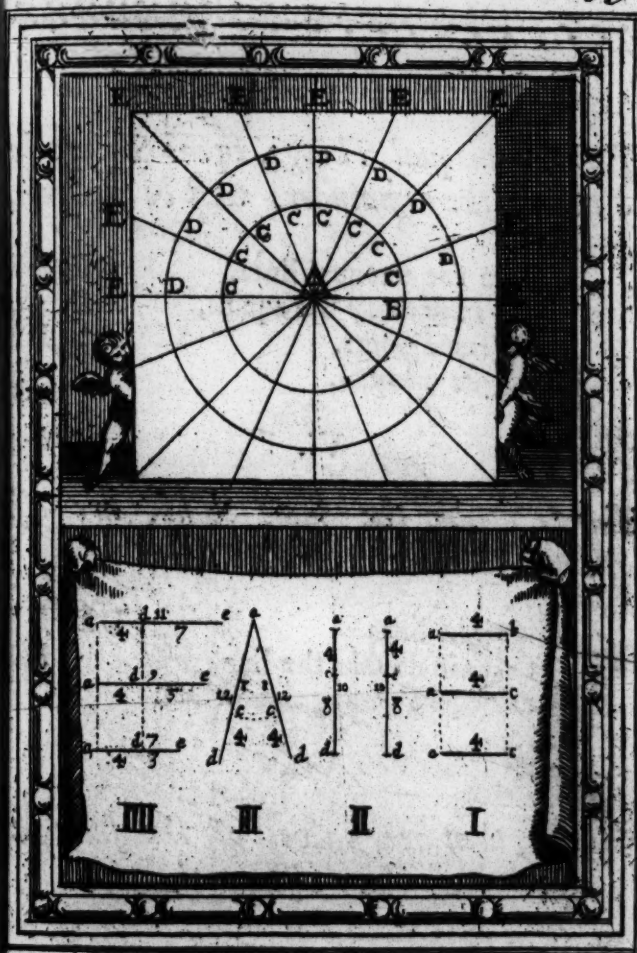
the whole

will be equal.

D E, D E

A D, A D

A E, A E





V.

If from unequal Things, equal Things be taken, the remainders will be unequal,

If from the unequal lines,	AE, AE
you take away the equals,	AD, AD
the remainders,	DE, DE
will be unequal.	

VI.

Things double the same third, are also equal to one another.

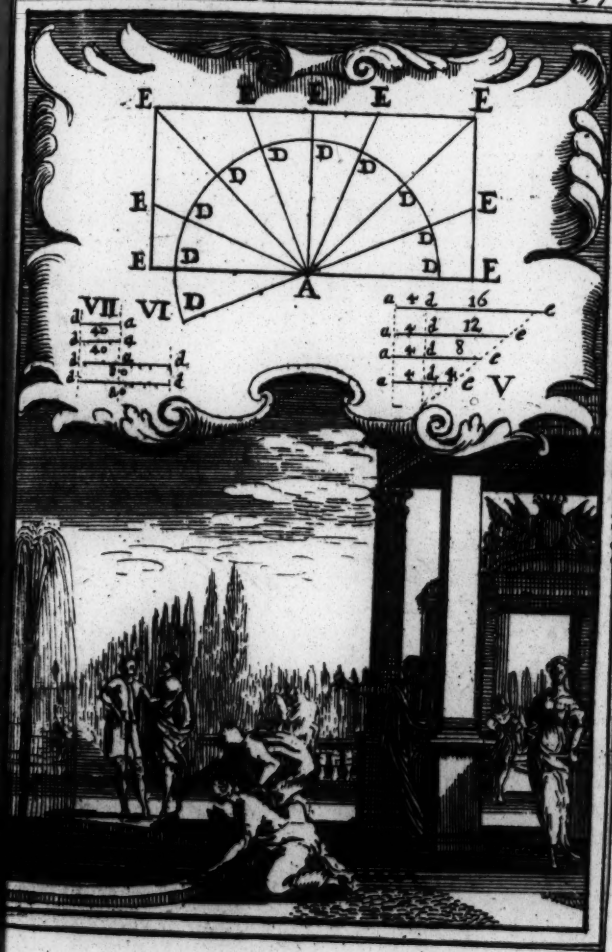
The right lines	DD, DD
that are double the line	AD
are equal among themselves,	

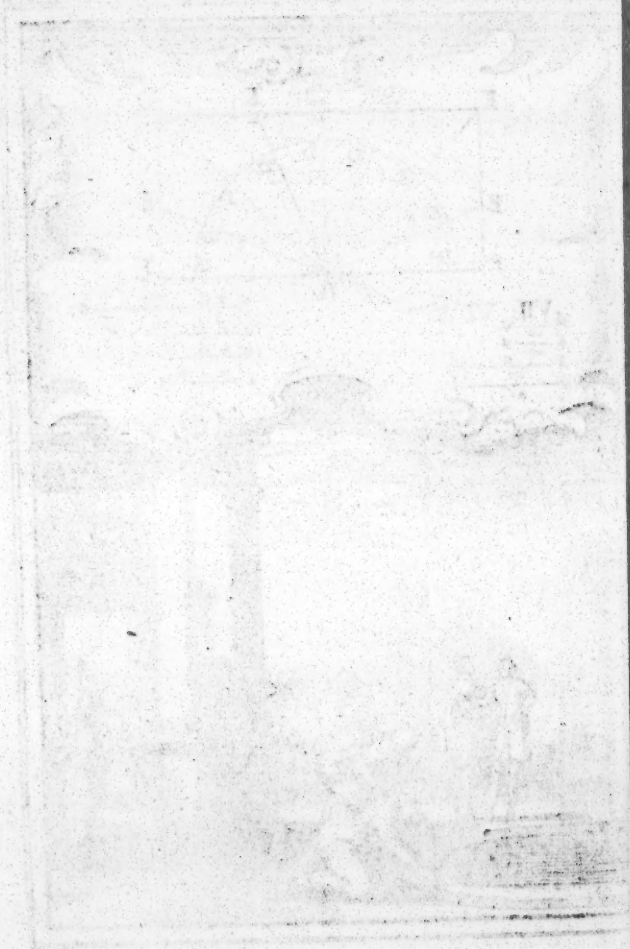
VII.

Things, that are halves of the same, or equal things, are also equal.

The lines	AD, AD
which are the halves of the lines DD, DD	
are equal to one another.	

What has been said of lines, may also be said of numbers, surfaces and bodies.





THE
PETITIONS.



THE

Petitions or Demands.

PETITION I.

D *Draw a right line from the point A
to the point B*

OPERATION.

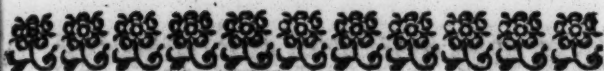
Apply a ruler to the points A & B
Draw the line demanded A B
by carrying the pencil along the Ruler, and
close to it from the point A
to the point B

PETITION II.

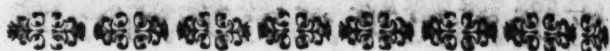
*Produce infinitely the line C D
on the side of the extremity D*

OPERATION.

Join the ruler to the line C D
Continue infinitely that line C D
on the side of the extremity D
by carrying the pen along, close to the ruler
towards E



Upon the point B describe the arc
 I point the point F draw the arc
 and the intersection of the two arcs will be



P E T I T I O N I I I .

*Describe a circle upon the point
and at the distance*

A B

O P E R A T I O N .

Set one of the points of the compass
upon the given point

Open the other to the given point

Turn the compasses about upon the point
and trailing the point

draw the circle demanded

B C D

P E T I T I O N I V .

On the points

E & F

make an intersection or section.

O P E R A T I O N .

Open the compasses at discretion, but so that the
distance of the two points of the compasses
may be greater than half the distance of the
points proposed

E & F

With this distance of the compasses

Upon the point E describe the arc

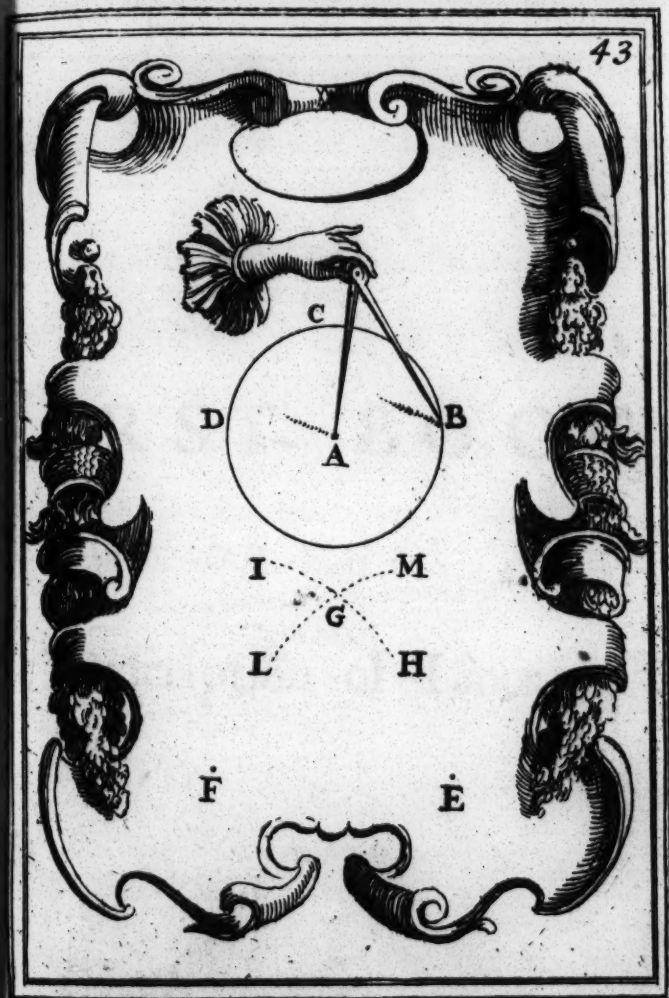
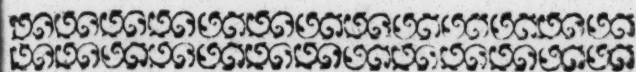
L M

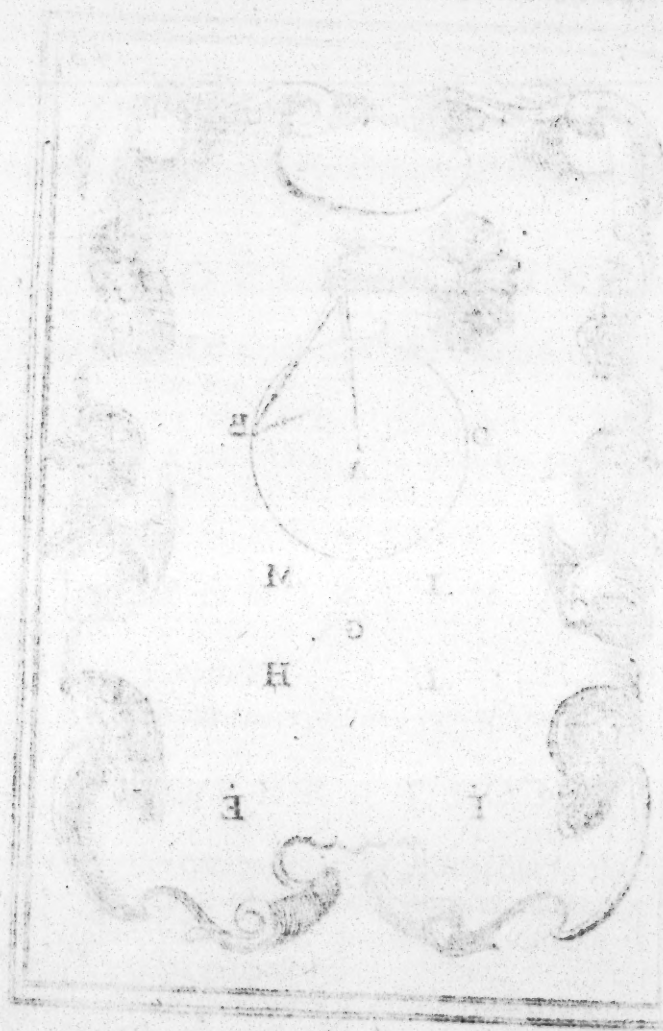
Upon the point F draw the arc

H I

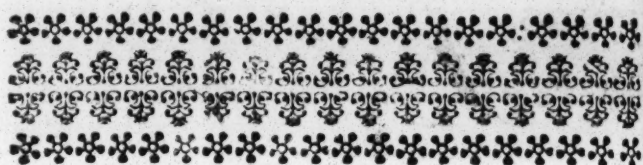
and the intersection requir'd will be

G





THE
FIRST BOOK
OF THE
Description of Lines.



BOOK the FIRST.

PROPOSITION I.

*To erect a perpendicular upon the middle
of a right line.*

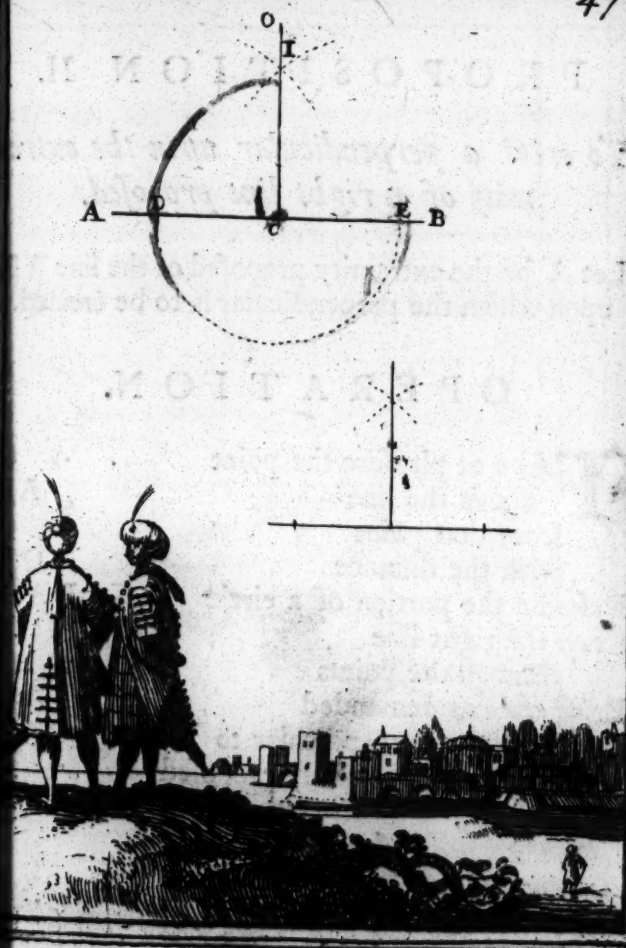
POSITION.

Let C be the point proposed in the middle of the
line A B, upon which the perpendicular is to
be erected.

OPERATION.

U	Pon the given point	C
	describe at pleasure the semicircle	D E
	upon the points	D & E
	make the section	I
	from the point	C
	draw the line demanded	C O
	thro' the section	I

This line C O will be perpendicular to the line
given A B, and erected upon the point pro-
posed C





P R O P O S I T I O N II.

To erect a perpendicular upon the extremity of a right line proposed.

Let A be the extremity proposed of the line AB, upon which the perpendicular is to be erected.

O P E R A T I O N.

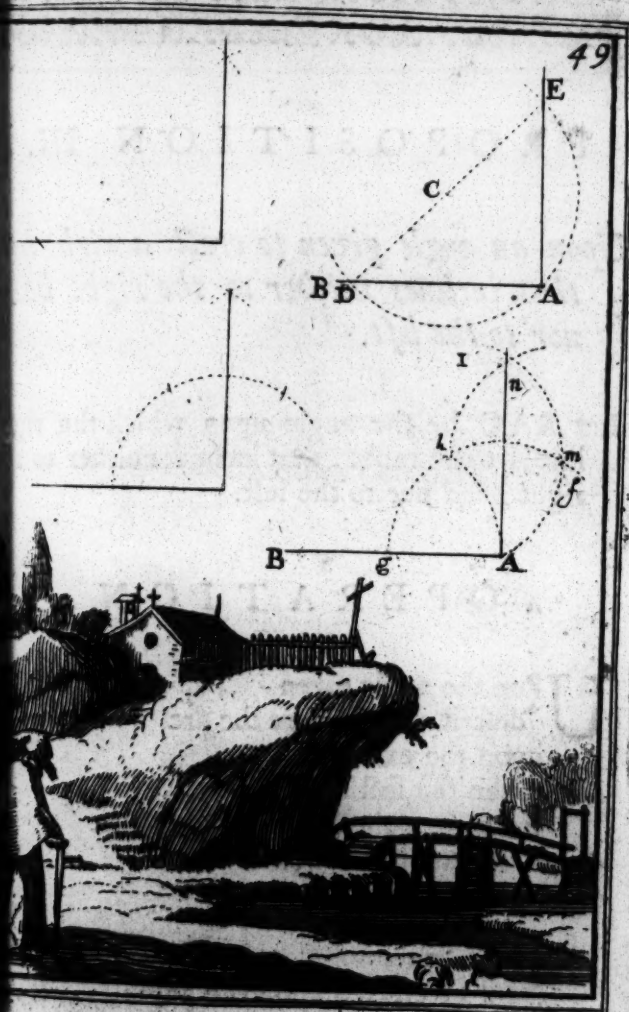
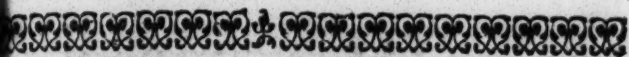
TAke at pleasure the point
 above the line
 from that point
 with the distance
 Describe the portion of a circle
 Draw the right line
 through the points
 Draw the line demanded
 it will be perpendicular to
 and at the extremity proposed

C
 AB
 C
 CA
 EAD
 DCE
 D&C
 AB
 AB
 A

Another way

Upon the point A describe the arc
 Upon the point g describe the arc
 Upon the point h describe the arc
 Upon the point m describe the arc
 Draw the line requir'd

g h m
 A h
 A m
 h n
 A n





PROPOSITION III.

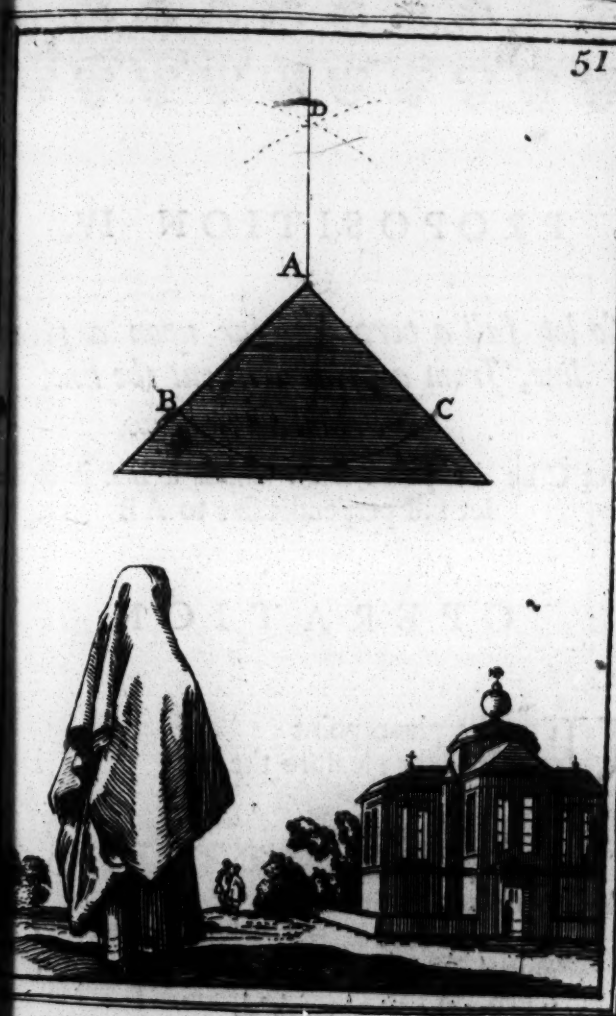
*Upon an angle given to erect a right line
that inclines neither to the right hand
nor to the left.*

Let BAC be the angle upon which the right
line is to be raised, that inclines neither to the
right hand nor to the left.

OPERATION.

UPon the angle given
describe at Pleasure the arc
upon the extremities
make the section
from the point of the angle given
draw the line requir'd
through the section.

This right line
shall be erected upon the angle
without inclining either to the right or left





PROPOSITION IV.

To let fall a perpendicular upon a given line, from a point without the line.

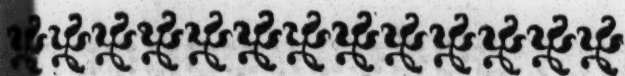
Let C be the point from which a line is to
let fall perpendicular to AB

OPERATION.

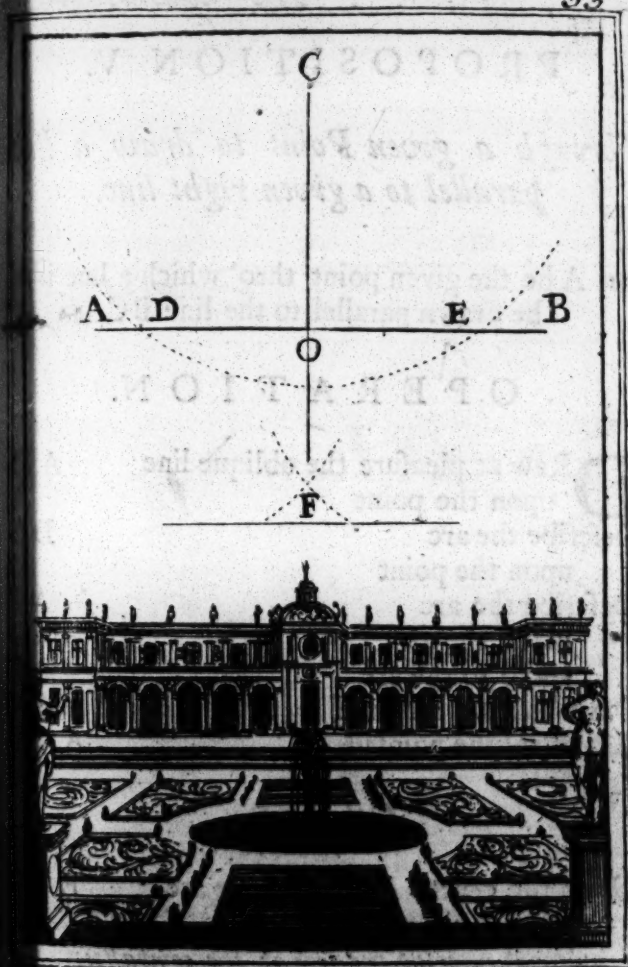
UPon the given point
describe at pleasure the arc
cutting the line
in the points
upon those points

As centres make the section
draw the line
and the line
will be the line requir'd.

D
A
D &
D &
C
C



53





PROPOSITION V.

*Through a given Point to draw a line
parallel to a given right line.*

Let A be the given point thro' which a line is to
be drawn parallel to the line BC.

OPERATION.

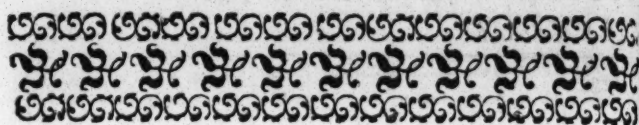
Draw at pleasure the oblique line
upon the point
Describe the arc
upon the point
Describe the arc
make the arc
equal to the arc
Draw the line required
thro' the points

A
D
A
D
A
M
A &

Otherwise.

Upon the centre A describe the arc
touching the line
without altering the legs of the compasses.
Upon the point H describe the arc
The point H is taken at pleasure in the line
Draw the demanded line
thro' the point
and touching the arc

E F
B
L R
B
O
L R



PROPOSITION VI.

To bissect a given finite right line.

POSITION.

Let A B be the right line proposed to be divided
into two equal parts.

OPERATION.

U Pon the extremity
as a centre, describe the arc

A
C D

*Without altering the distance of the legs of the
compasses.*

Upon the other extremity
as a centre, describe the arc

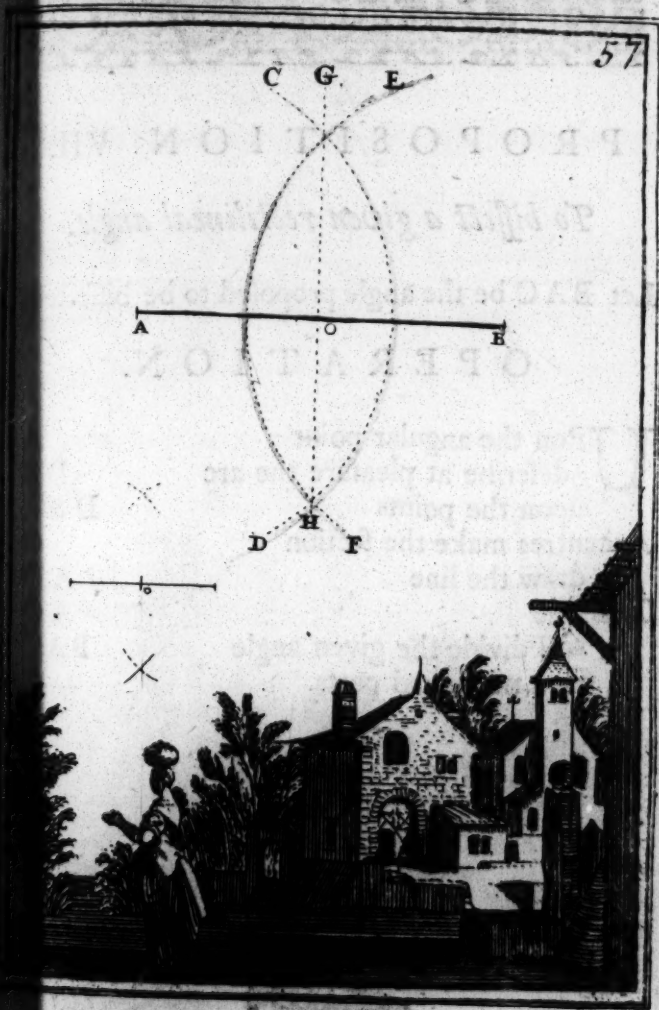
B
E F

*These arcs are to be made so as to intersect each
other.*

Draw the right line
through the intersections

G H
G & H
O

A B then will be bisected at the point





P R O P O S I T I O N VII.

To bissect a given rectilineal angle.

Let BAC be the angle proposed to be bissected

O P E R A T I O N.

UPon the angular point ·
describe at pleasure the arc
upon the points

As centres make the section
draw the line

This line

will divide the given angle
into two equal parts.

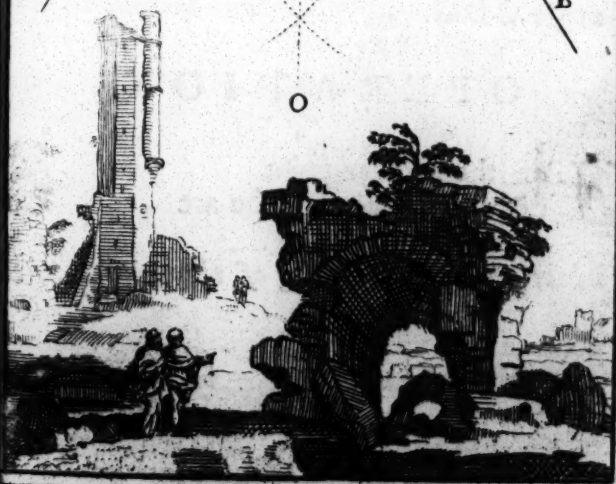
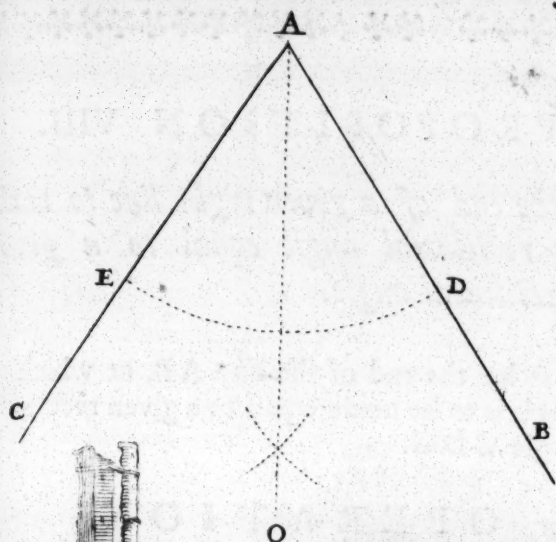
3

A
DE
D & E
C
AC
AC
BA





59





PROPOSITION VIII.

At the end of a given right line to make a rectilineal angle equal to a given rectilineal angle.

Let A be the end of the line AB, at which an angle is to be made equal to a given rectilineal angle CDG.

OPERATION.

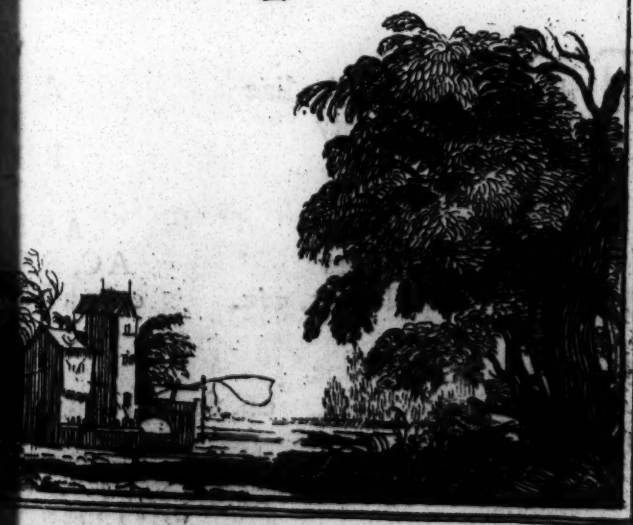
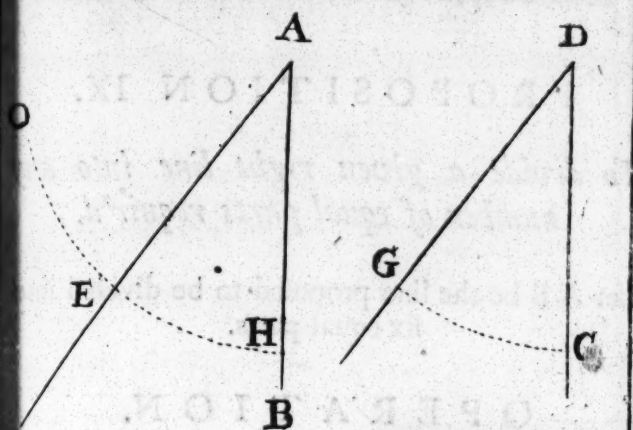
U Pon the angular point D
describe at pleasure the arc CG

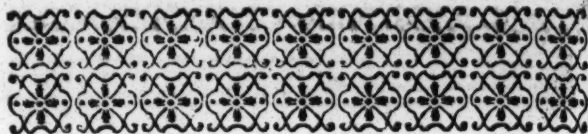
Without altering the opening of the compasses.

Upon the extremity	A
describe the arc	HO
Make the arc	HE
equal to the arc	CG
draw the line	AE
The angle	BAE
will be equal to the angle	CDG
which was the Thing proposed.	

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

61





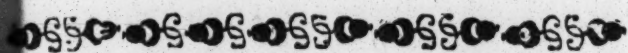
PROPOSITION IX.

To divide a given right line into any number of equal parts requir'd.

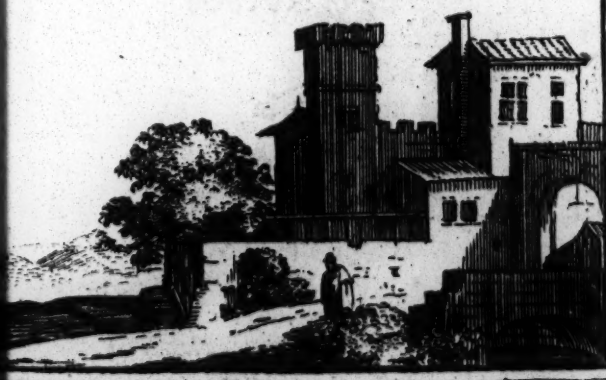
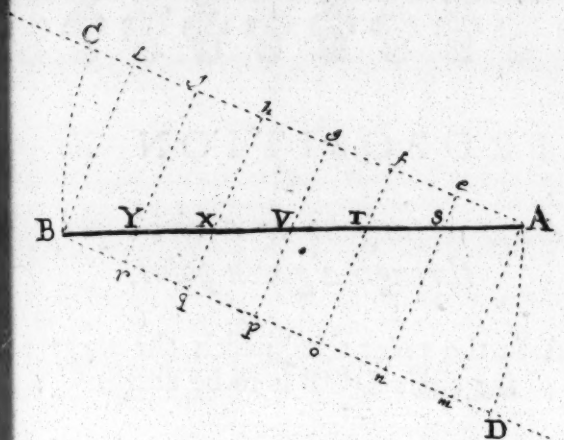
Let A B be the line propos'd to be divided into six equal parts.

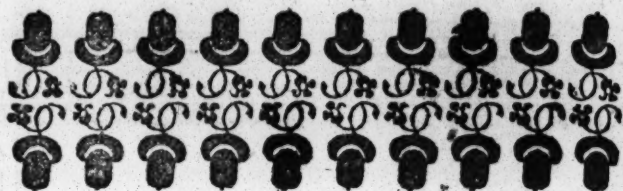
OPERATION.

F rom the point	A
draw at pleasure the line	A C
thro' the extremity	B
Draw the line	B D
parallel to the line	A C
from the points	A & B
and along the lines	A C, B D
Carry any six equal parts, viz.	e f g h I L
along the line	A C
Rqpon m along the line	B D
draw the lines	en, fo, gp, hq, I R
Then the line	A B
will be divided into six equal parts at the	S, T, V, X, Y
sections	



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PROPOSITION X.

*To draw a tangent to a circle proposed
through a given point.*

Let A be the point thro' which the tangent to
the circle DOP is to be drawn.

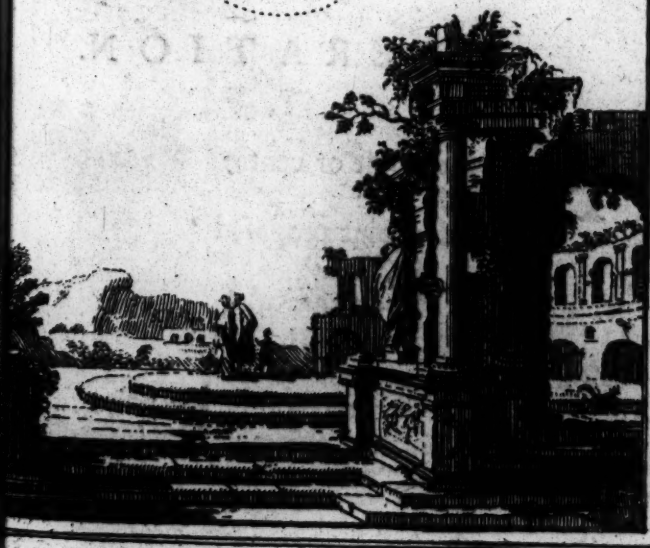
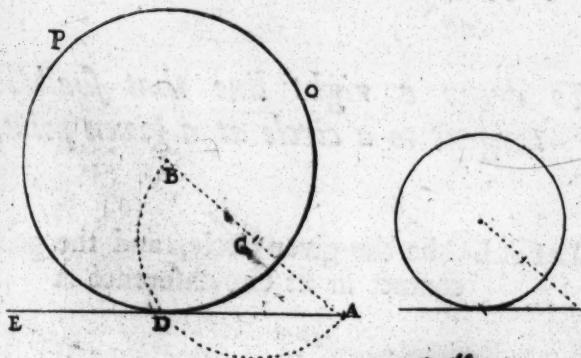
OPERATION.

From the centre of the circle
draw the secant
divide the line
into two equal parts in
upon the point
with the radius
Describe the semicircle
cutting the circle in
from the given point
Draw the right line
thro' the point
This right line
will be the tangent requir'd.

B
BA
BA
C
C
CA
ADB
D
A
AB
D
AB



65



F



PROPOSITION XL.

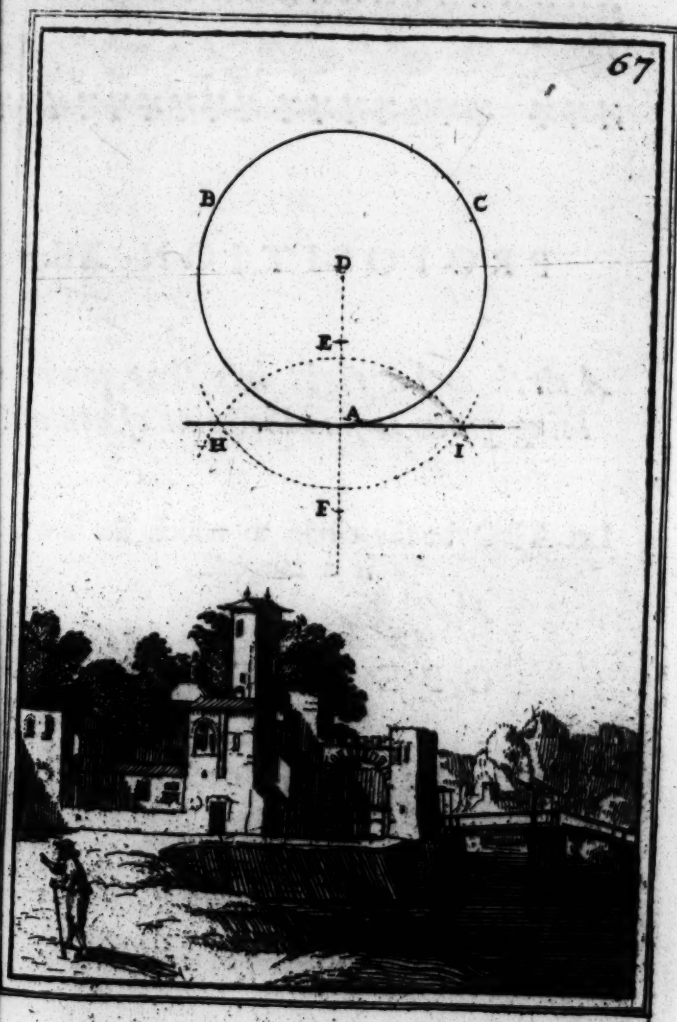
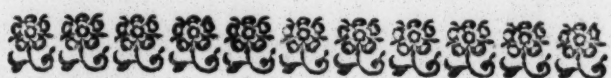
To draw a right line that shall be tangent to a circle at a given point.

Let ABC be the given circle, and the point of contact in its circumference A

OPERATION.

From the point or centre draw the line thro' the point proposed and to the line draw the perpendicular continued towards

This tangent will touch the circle at the point which was the thing required.





PROPOSITION XII.

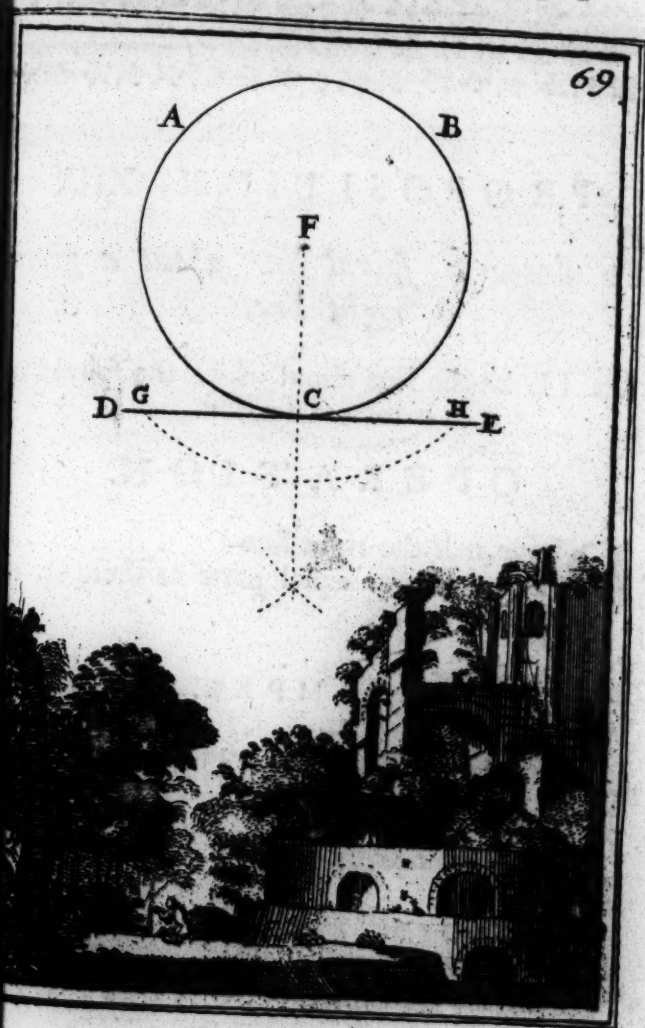
*A circle and a right line that touches it
being given to find the point of contact.*

Let ABC be the circle to which the line GE
is a tangent.

OPERATION.

Pag. 8. **F**rom the centre of the circle
let fall the perpendicular
upon the tangent

The section
will be the point of contact sought.





PROPOSITION XIII.

To draw a spiral line about a given right line.

Let IL be the line about which the spiral line is to be described.

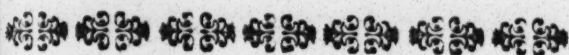
OPERATION.

Pag. 18. **D**ivide half the right line IL into as many equal parts as there are to be revolutions.

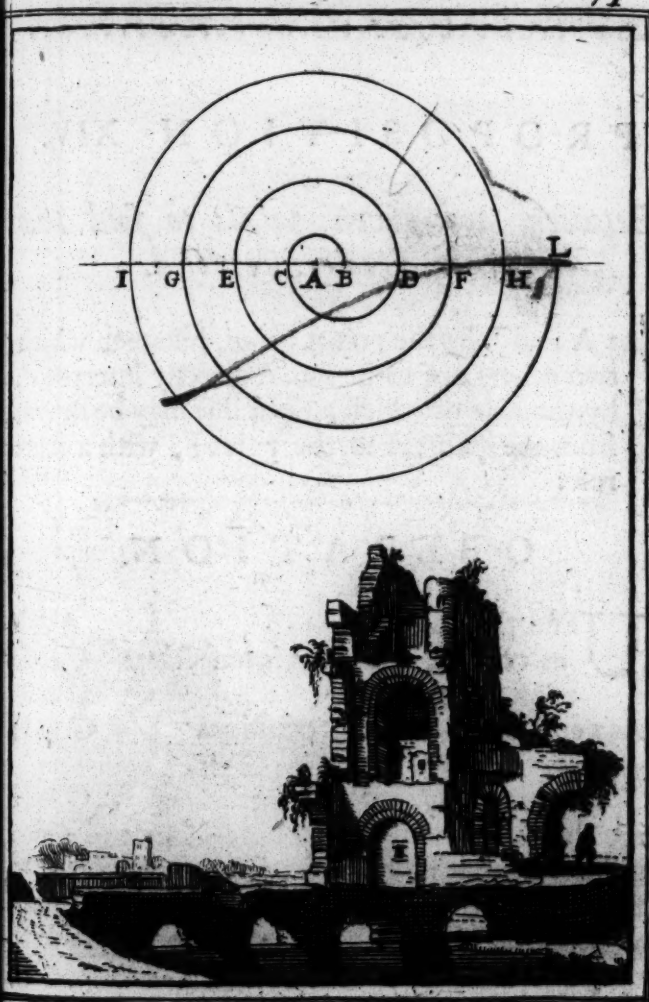
EXAMPLE.

To make one of four revolutions.

Pag. 11. Divide the half
 into four equal parts
 Divide also
 into two equal parts in
 upon the point
 Describe the Semicircles BC, DE, FG, HI
 upon the point
 Describe the Semicircles CD, EF, GH, IL
 and you will have the spiral line fought.



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PROPOSITION XIV.

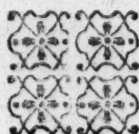
Between two given points to find two other directly interposed.

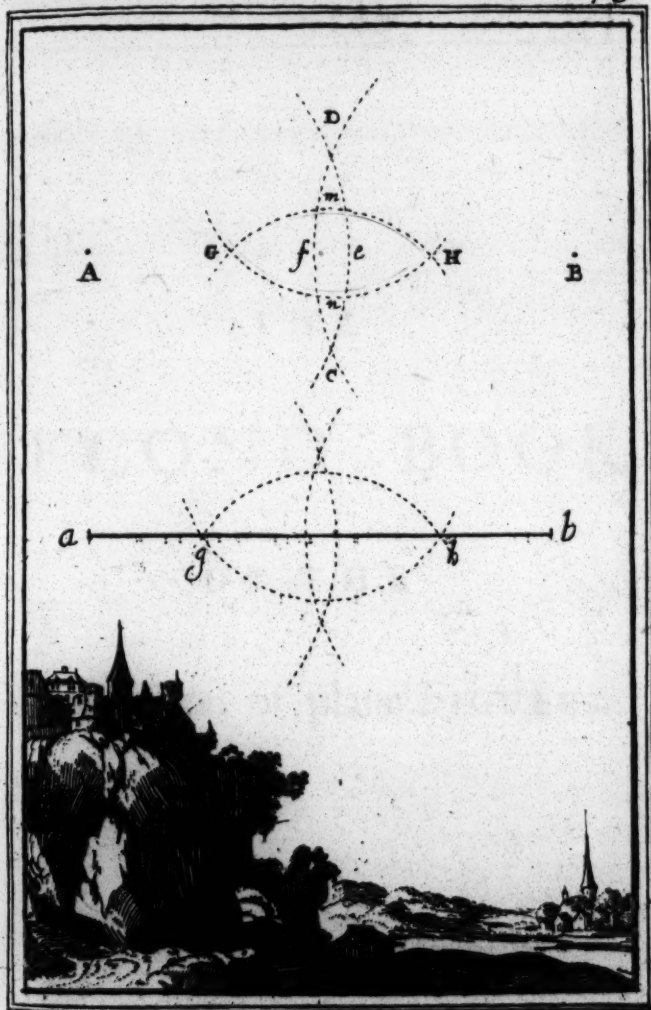
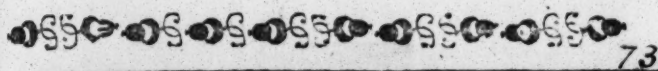
Let A and B be the points given, between which two others are to be found directly interpos'd, by the help of which a right line may be drawn from the point A to the point B, with a short rule.

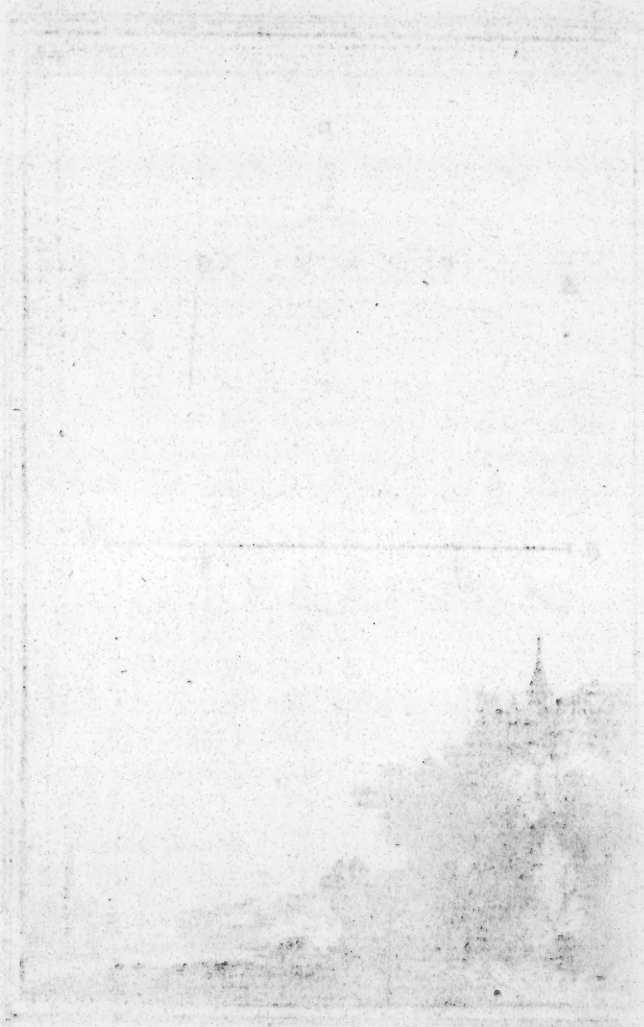
OPERATION.

U	Pon the points	A & B
	as centres, make the interfections	C & D
	upon the points	C & D
	As centres make the interfections	G & H

These points G & H are the points requir'd, by the assistance of which a right line may be drawn from the point A to the point B, which could not be done at once with a rule less than the length between A & B.







THE
SECOND BOOK

OF THE

Construction of plane FIGURES.



BOOK the SECOND.

PROPOSITION I.

To make an equilateral triangle upon a given line.

Let AB be the given line upon which the equilateral triangle is to be constructed.

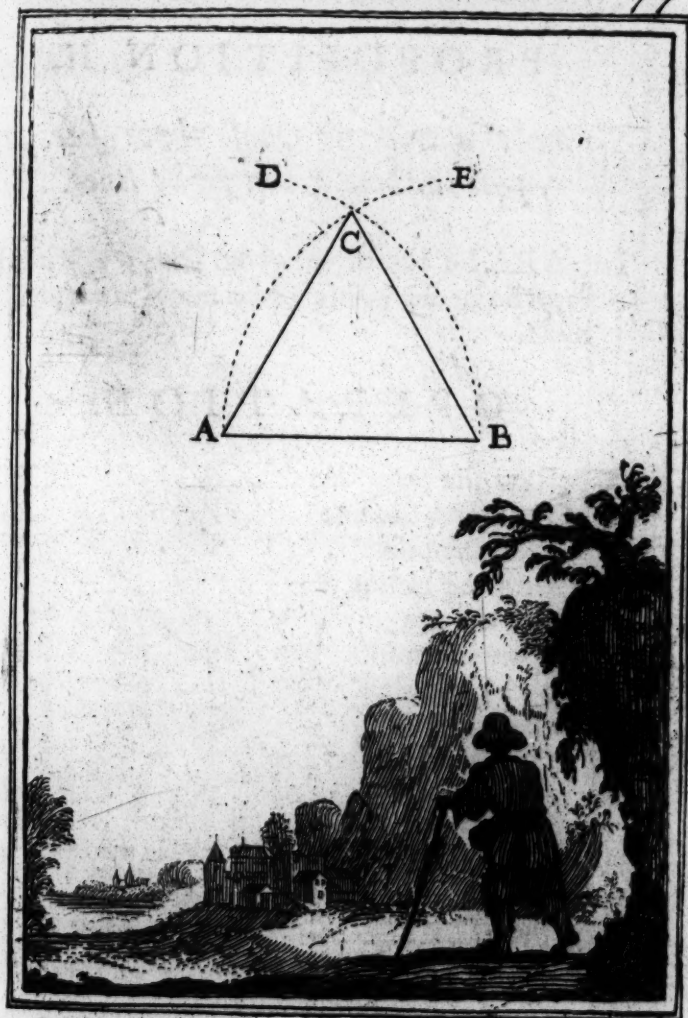
OPERATION.

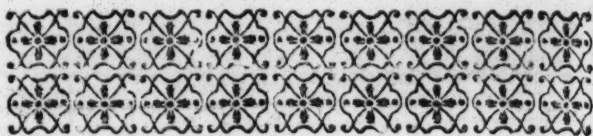
UPon the extreme point	A
with the radius	AB
Describe the arc	BD
upon the extremity	B
with radius	BA
Describe the arc	AE
from the intersection	C
Draw the lines	CA, CB

ABC will be the equilateral triangle required.



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PROPOSITION II.

To make a triangle whose three sides are equal to three given right lines.

Let ABC be the three given lines; a triangle is to be made whose three sides are equal to them.

OPERATION.

D Raw the right line
equal to the line
upon the point
with the radius

Describe the arc
upon the point
with the radius

Describe the arc
from the interfection

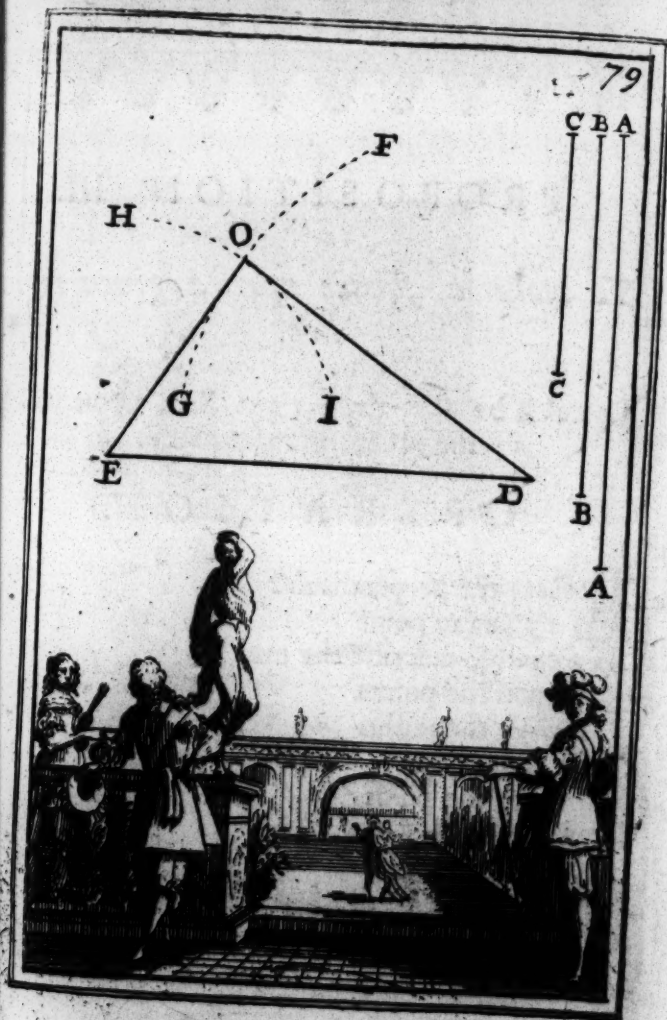
Draw the lines

DE
AA
D
BB
GF
E
CC
HI
O
OE, OD

The triangle
will be composed of three sides equal to the three
lines given

DEO

AA, BB, CC.





PROPOSITION III.

To make a square upon a given right line.

Let AB be the given right line, upon which the square is to be made.

OPERATION.

Pag. 50.

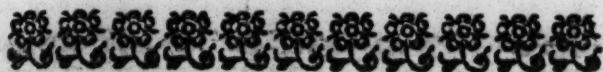
E Rect the perpendicular
upon the point
As a centre, describe the arc
upon the points
with the radius
Make the section
from the point
Draw the lines

A C
A
B C
B & C
A B
D
D
D C, D B

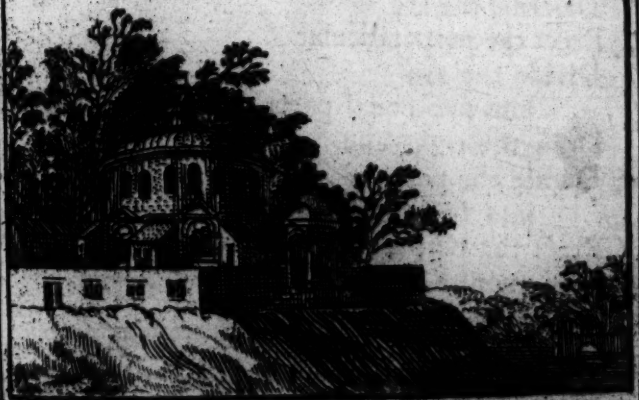
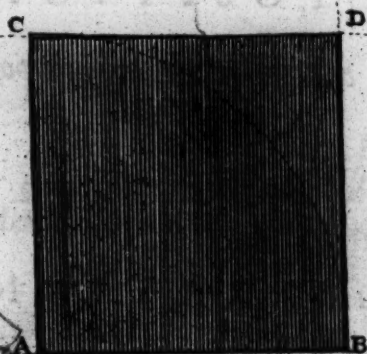
ABCD will be the square requir'd to be constructed upon the given right line

A B





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PROPOSITION IV.

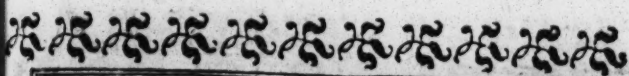
To make a regular pentagon upon a given right line.

Let A B be the given line, upon which the pentagon is to be constructed.

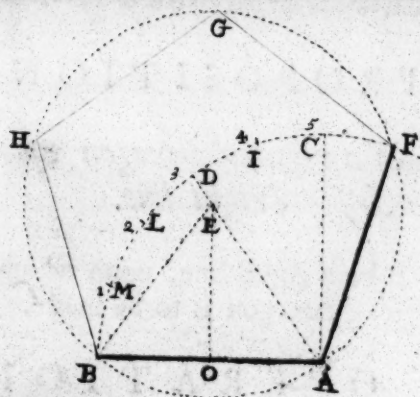
OPERATION.

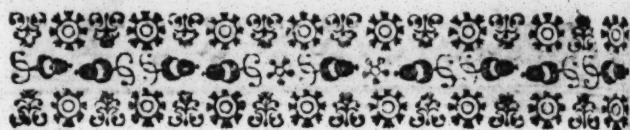
- U**pon the extremity
and with the radius
Describe the arc
Pag. 50. Erect the perpendicular
Divide the arc
into five equal parts
Draw the right line
Pag. 58. Divide the base
into two equal parts in
Pag. 46. Erect the perpendicular
upon the intersection
with the radius
Describe the circle
Carry round five times, the line
in the circumference of the circle, and a regular equiangular equilateral pentagon, will be completed.

A
A B
B D F
A C
B C
I D L M
A D
A B
O
O E
E
E A
A B F G H



83





PROPOSITION V.

To make a regular hexagon upon a given right line.

Let A B be a right line, upon which a regular hexagon is to be made.

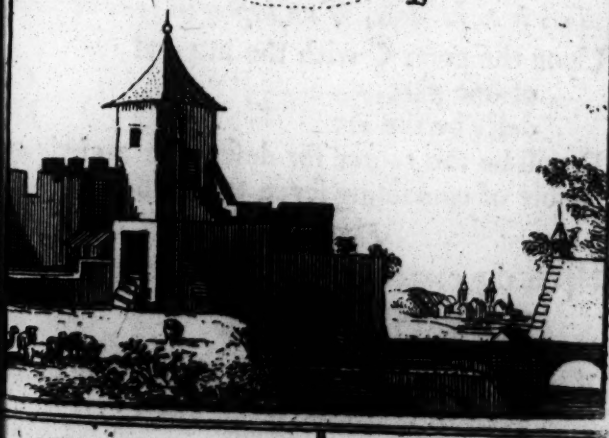
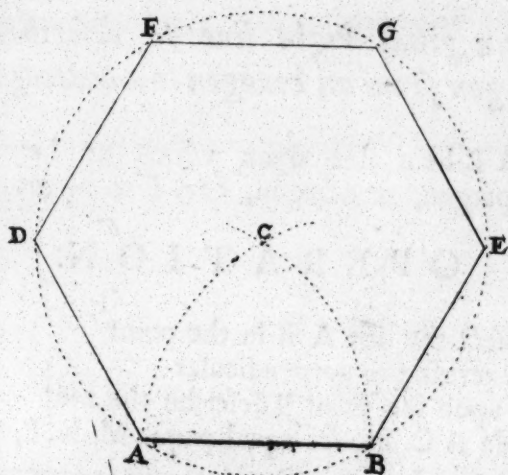
OPERATION.

U	Pon the extremities	A & B
	and with the radius	A B
	Describe the arcs	A C, B C
	upon the section	C
	Describe the circle	A B E F G
	Carry six times the line given	A B
	in the circumference and you will have a	A B E F G D
	regular hexagon	A B
	upon the given line	





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PROPOSITION VI.

Upon a given right line to describe any polygon from an hexagon to a dodecagon.

Let A B be a line upon which an hexagon, heptagon, or octagon, &c. is to be made.

OPERATION.

Pag. 58. **B** Ifect the line A B in the point

Pag. 46. **B** erect the perpendicular

upon the point B describe the arc

Divide A C into fix equal parts M, N, P, Q, R

This is to be done, if an heptagon is to be made.

Upon the point C with the interval
of one part

describe the arc

D will be the center for describing a circle capable of containing seven times the line given

For an octagon.

Upon the center C, with the interval
of two parts

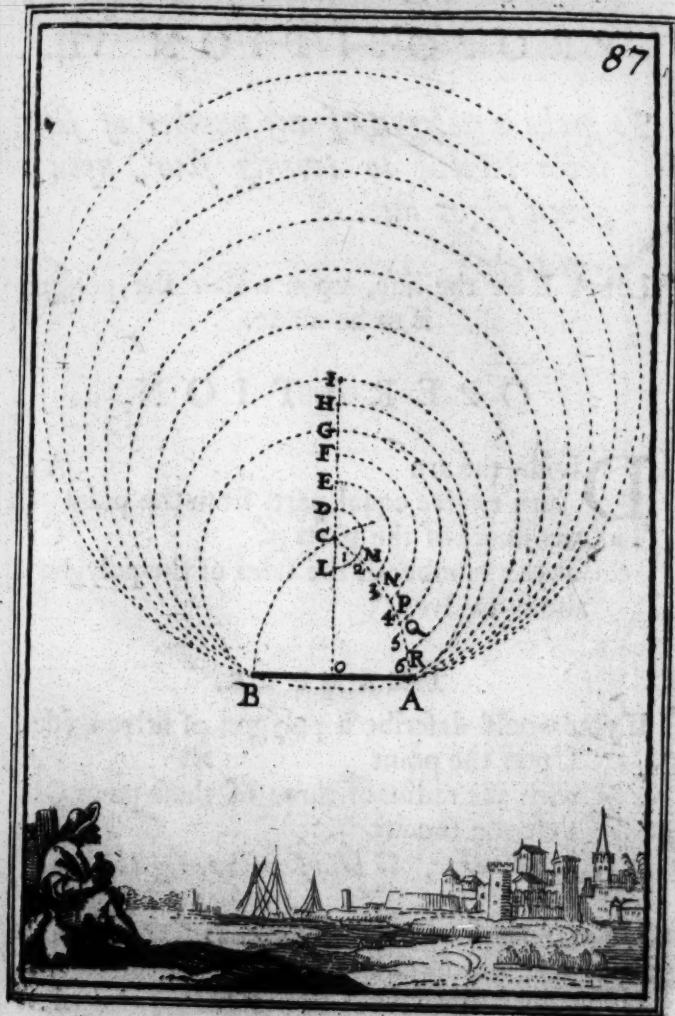
Describe the arc

E will be the center of a circle capable of containing eight times the given line

For an heptagon.

Take three parts

and so for the rest adding one part.





PROPOSITION VII.

To make a polygon of any number of sides from twelve to twenty four, upon a given right line.

Let A B be the line, upon which the polygon is to be made.

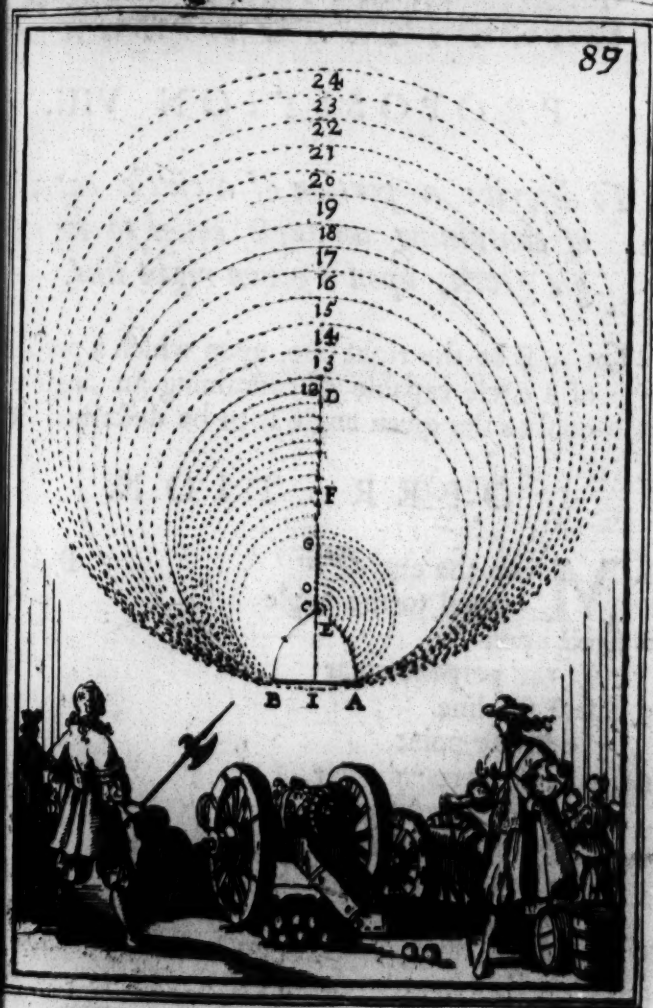
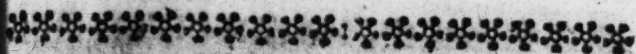
OPERATION.

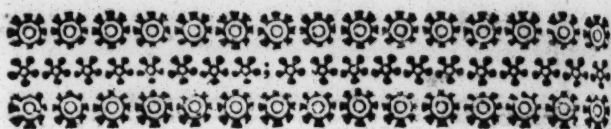
Divide the arc A C
 into twelve equal parts from the point C
 Take as many of the parts of C A
 as the number of the sides of the polygon is
 above twelves.

EXAMPLE.

If you would describe a polygon of fifteen sides,
 Upon the point C
 with the radius of three of these parts C E
 describe the arc E O
A C of twelve, C O of three together make fifteen.
 Upon the point O with the radius O B
 describe the arc B F
 Upon the point F with the radius F A
 describe a circumference, and it will contain
 the line given A B
 twelve times.

and so also for any other polygon.





PROPOSITION VIII.

To describe a portion of a circle capable of containing an angle equal to an angle given, upon a given right line.

Let A B be the right line, upon which a portion of a circle capable of containing an angle equal to the given angle is to be described C

OPERATION.

Pag. 62. **M**Ake the angle equal to the angle

Pag. 50. Erect upon the perpendicular

Pag. 58. Bisect the line in the point

Pag. 46. Erect the perpendicular upon the section with the radius

Describe the portion of the circle

All the angles you make in this segment of the circle, and upon the given line will be equal to the angle

B A D

C

A D

A E

A B

H

H F

F

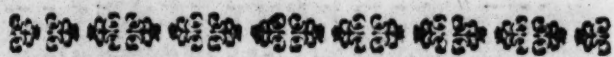
F A

A E B

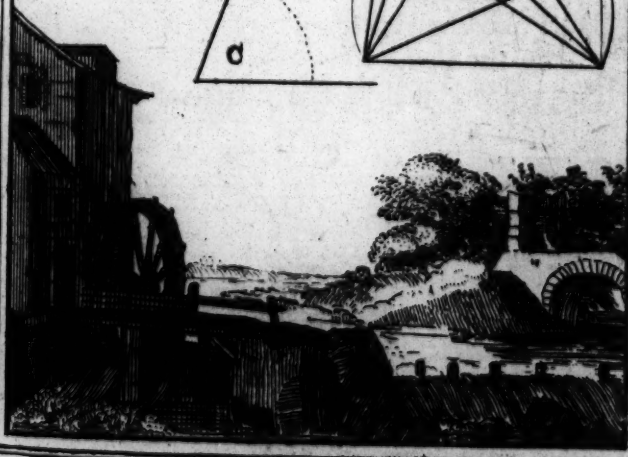
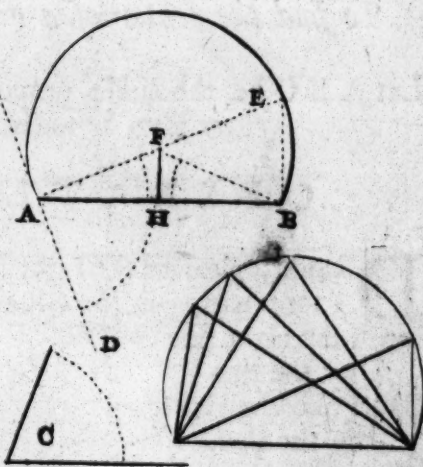
of the

A B

C



91.





PROPOSITION IX.

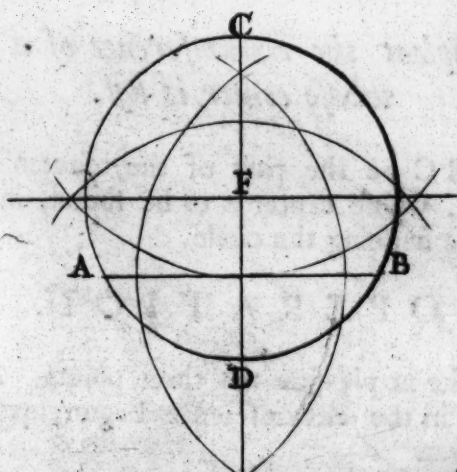
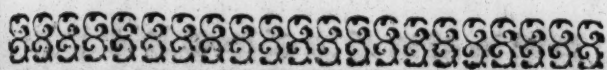
To find the center of a given circle.

Let A B C be the circle proposed, whose center is to be found.

OPERATION.

	D raw at pleasure the right line	A B
	terminating in the circumference	A B C
Pag. 58.	Bisect the right line	A B
	by the line	D C
Pag. 58.	Bisect also the right line	C D
	in the point	F
	The point F will be the center of the circle re-	
	quir'd	A B C







PROPOSITION X.

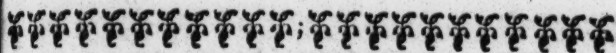
*To compleat the circumference of a circle
whose center is lost.*

Let A B C be the part of the circumference
given, whose center is to be found, in order
to the finishing the circle.

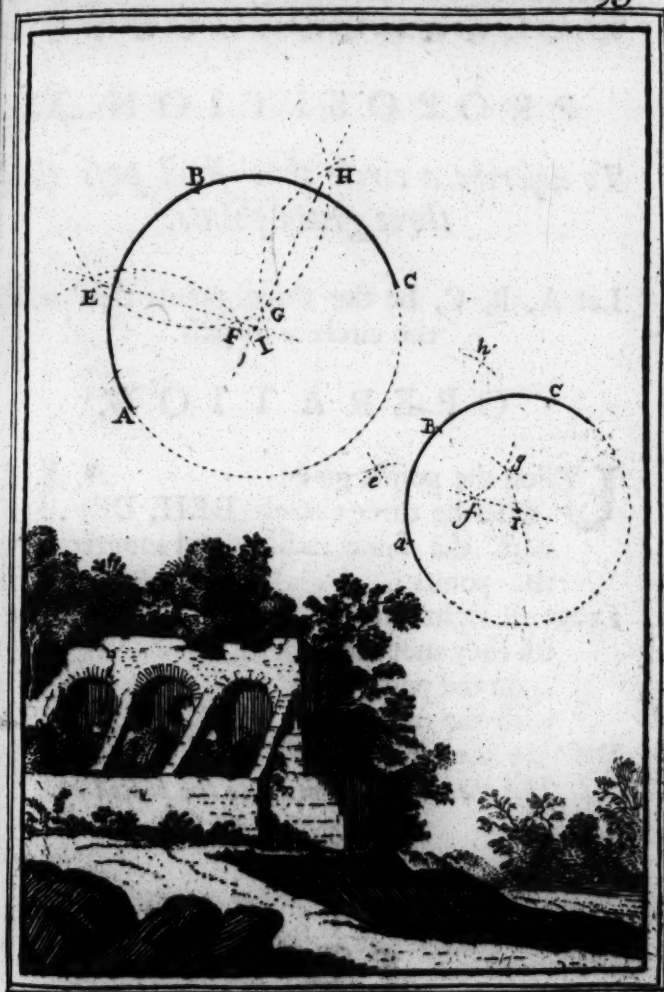
OPERATION.

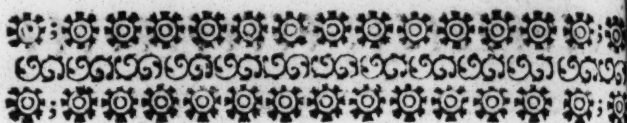
T Ake at pleasure the three points	A B C
in the circumference begun upon the	
points	A & B
Make the sections	E & F
Draw the right line	E F
upon the points	B & C
Make the sections	G & H
draw the right line	G H
upon the interfection and center	I
and with the interval	I A
compleat the circumference begun.	





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P R O P O S I T I O N X I.

To describe a circle that shall pass thro' three given points.

Let A, B, C, be the three points thro' which the circle is to pass.

O P E R A T I O N.

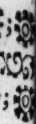
U Pon the points given A, B, C,
describe three circles DEH, DEF, FGL
with the same radius, and intersecting at
the points D & E, F & G

Draw the right lines DE, FG
till they meet in I
upon the point I
with the radius IA

Describe the circle requir'd.

This operation is similar to the preceding.





I.

ro'

rich

C.

GL

at

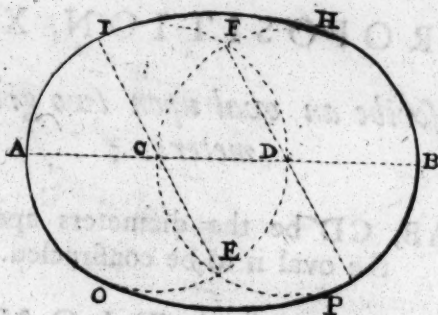
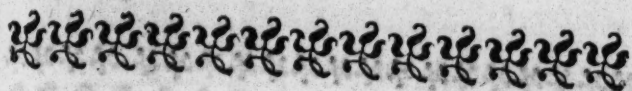
G

G

I

I

A





PROPOSITION XIII.

To describe an oval upon two given diameters.

Let AB, CD be the diameters upon which the oval is to be constructed.

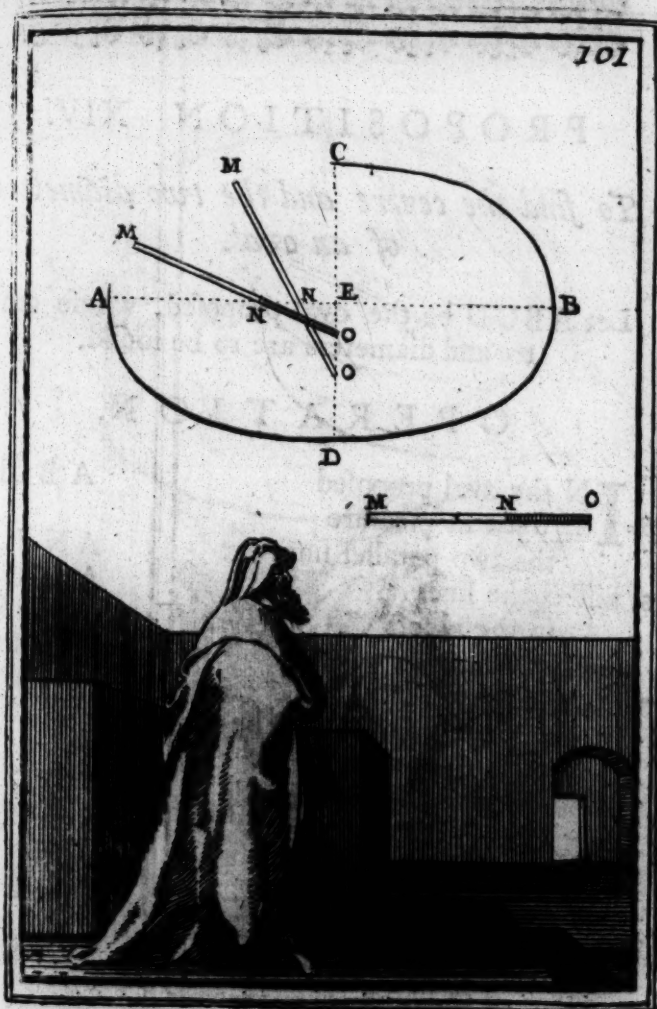
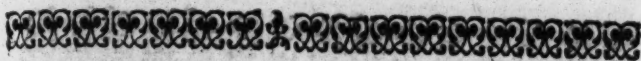
OPERATION.

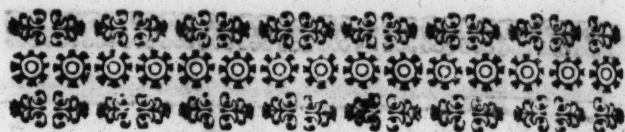
MAke the ruler
 equal to the greater semi-diameter AE
 upon which mark the length MN
 equal to the lesser semi-diameter CE

This Ruler being thus dispos'd,

Place it after such a manner upon the diameters

AB, CD
 that the point N
 sliding along the line AB
 the extremity O
 may always be in the line CD
 carrying along thus the rule MO
 Describe the oval with the extremity M





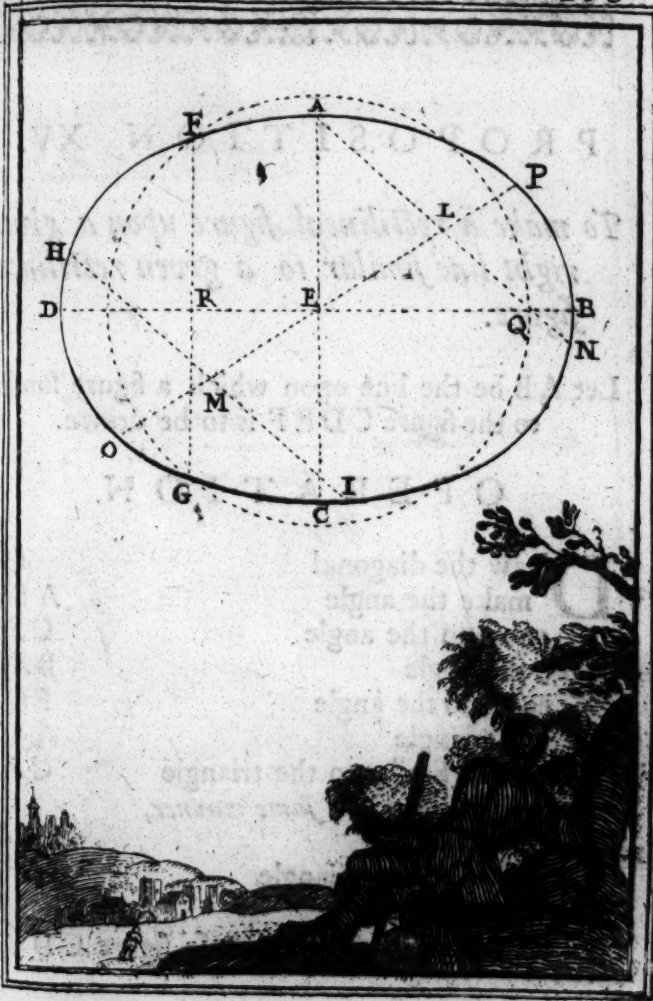
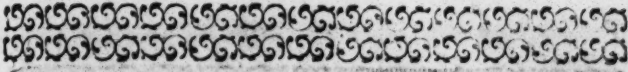
PROPOSITION XIV.

To find the centre and the two diameters of an oval.

Let ABCD be the oval proposed, whose centre and diameters are to be found.

OPERATION.

	I N the oval proposed	A B C D
Pag. 56.	draw at pleasure the two parallel lines	A N, H I
Pag. 56.	Bisect the lines in the points	A N, H I
	draw the line	L & M
Pag. 58.	Bisect it in	P L M O
	and the point E will be the centre upon the point	E
	Describe at pleasure the circle cutting the oval in	E
	thro' the intersections	F G Q
	Draw the right line	F & G
Pag. 58.	Bisect it in	F & G
	Draw the greatest diameter	F G
	thro' the points	R
	thro' the centre	B D
Pag. 56.	Draw the least diameter	E R
	parallel to the line	E
	and what was proposed will be effected.	A E C
		F G





PROPOSITION XV.

To make a rectilineal figure upon a given right line similar to a given rectilineal figure.

Let AB be the line upon which a figure similar to the figure CDEF is to be drawn.

OPERATION.

Pag. 62. **D**raw the diagonal
make the angle
equal to the angle

Pag. 62. Make the angle
equal to the angle
the triangle
will be similar to the triangle
after the same manner,

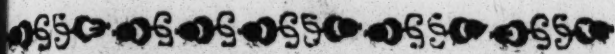
Pag. 62. Make the triangle
similar to the triangle

The whole figure
will be similar to the whole figure CDEF

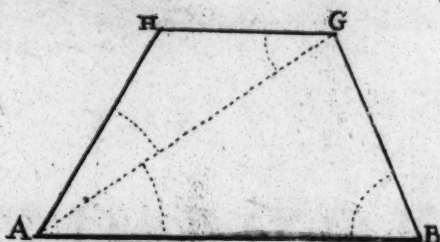
CE
ABG
CFE
BAG
FCE
ABG
CFE

AGH
CED
ABGH
CDEF

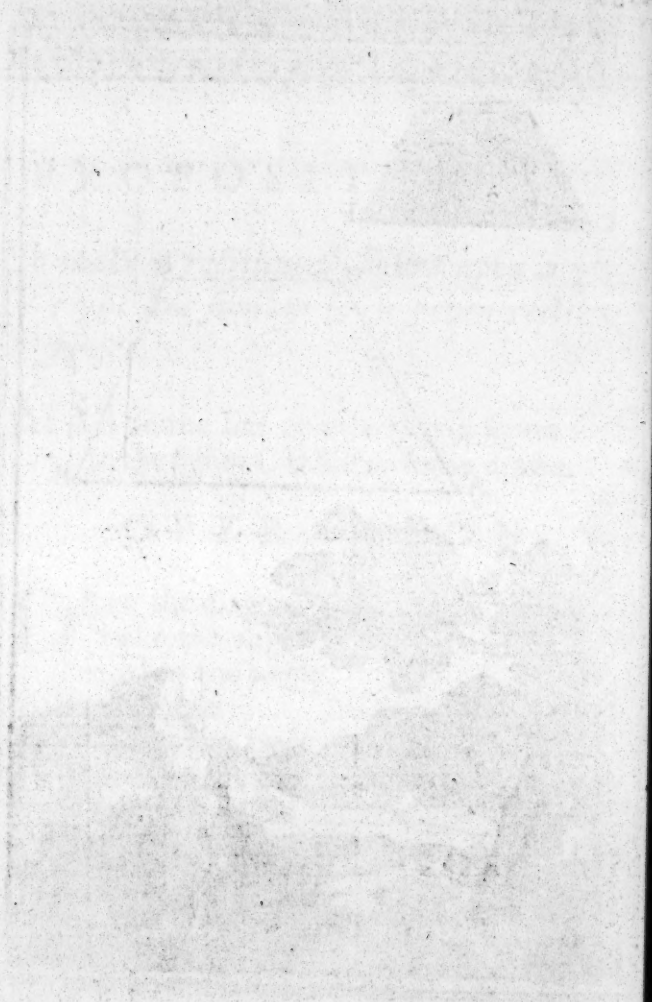




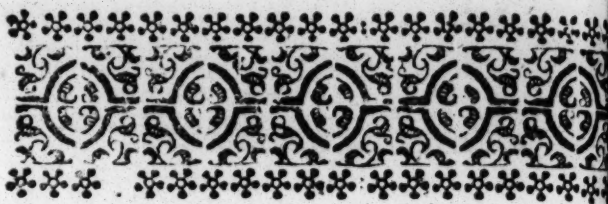
105



PROBLEM. To find the Area of a Triangle.



THE
THIRD BOOK,
OF THE
Inscribing of FIGURES.



BOOK the THIRD

PROPOSITION I.

To inscribe in a given circle, an equilateral triangle, hexagon or dodecagon.

Let A C D be the circle in which an equilateral triangle, &c. is to be inscribed.

OPERATION.

For an equilateral triangle.

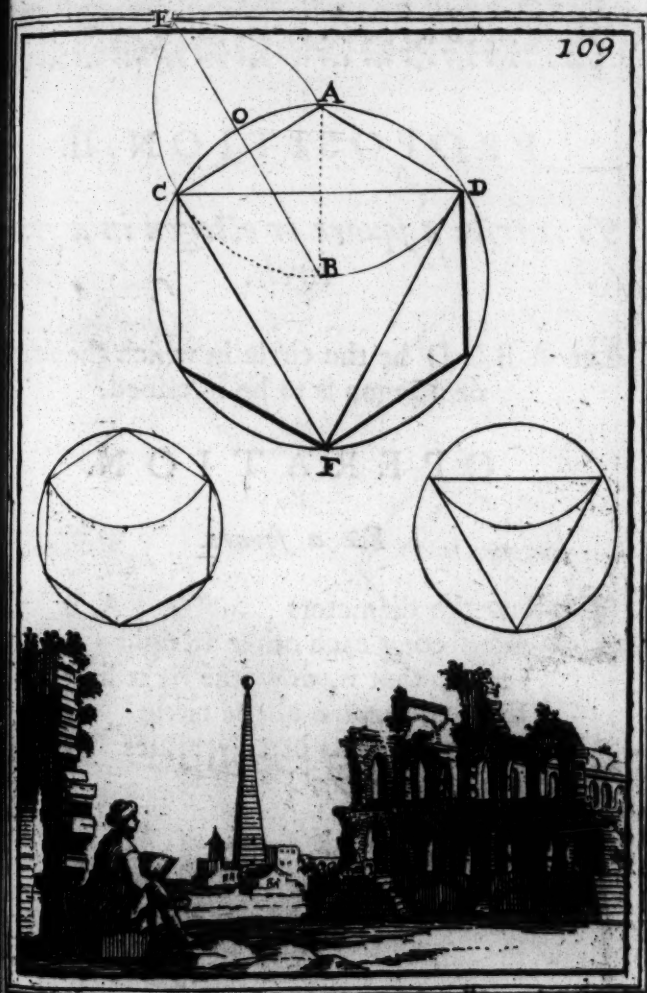
UPON a point as
with the interval of the semidiameter A
Describe an arc C B
Draw the right line D
Carry that distance C
from the point
to the point
Draw the lines F C, F
The triangle required will be C D

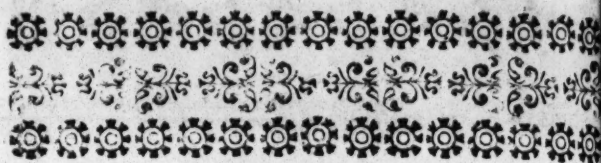
For an hexagon

Carry round six times the semidiameter A
in the given circumference

For a dodecagon

Pag. 58. Bisect the arc of the hexagon A
in the point
the side of the dodecagon will be A





PROPOSITION II.

To inscribe a square or octagon in a given circle.

Let A B C D be the circle in which the square or octagon is to be inscribed.

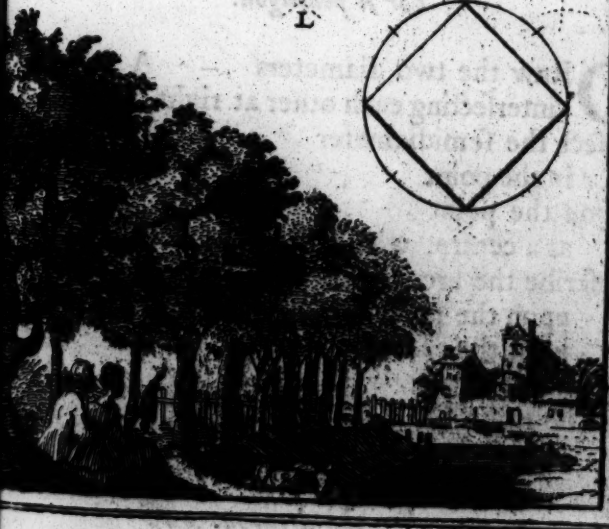
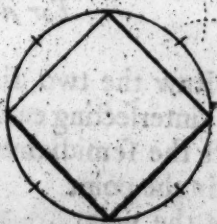
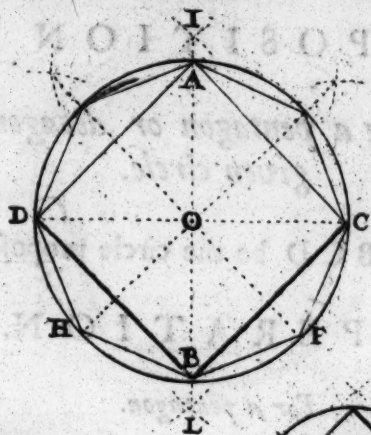
OPERATION.

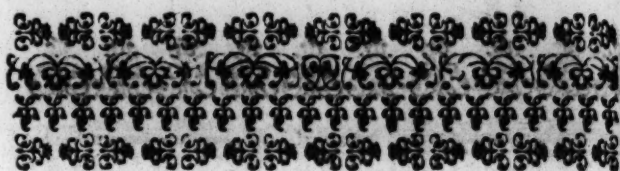
For a square.

Draw the diameters A B, C D intersecting each other at right angles, that is, draw the right line thro' the centre of the circle upon the points or extremities C & D.
 Make the intersections I & H.
 Draw the right line passing thro' the centre O.
 These lines or diameters A B, C D will intersect at right angles.
 Draw the lines AC, AD, BC, BD, & ACBD will be the square requir'd.

For an octagon.

Fig. 58. Subdivide each quarter of the circle into two equal parts, and you will have an octagon.





P R O P O S I T I O N I I I .

To inscribe a pentagon or decagon in a given circle.

Let A B C D be the circle proposed.

O P E R A T I O N .

For a pentagon.

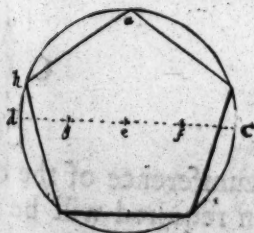
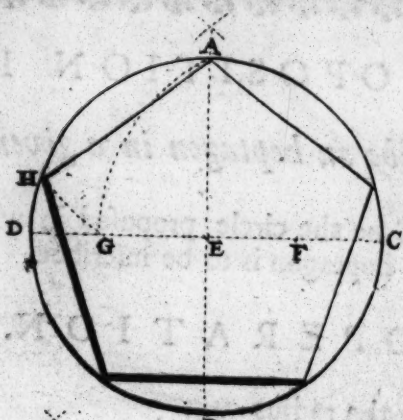
Pag. 58.	D Raw the two diameters	A B, C D
	intersecting each other at right angles in E	
	bisect the semidiameter	C E
	in the point	F
	Upon the point	F
	as a centre, with the radius	F A
	Describe the arc	A G
	upon the point	A
	with the radius	A G
	Describe the arc	G H
	The right line	A H
	will divide the circle into five equal parts,	

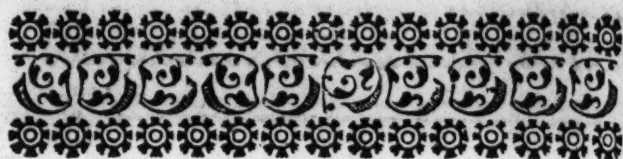
For the decagon.

Pag. 58. Subdivide each part of the circle into two equal parts.



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PROPOSITION IV.

To inscribe an heptagon in a given circle.

Let A B C be the circle proposed in which the heptagon is to be inscribed.

OPERATION.

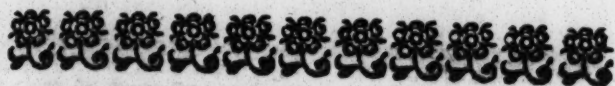
Draw the radius
upon the extremity
with the radius
Describe the arc
Draw the right line
Carry the half

I A
A
A I
C I C
C C
C O

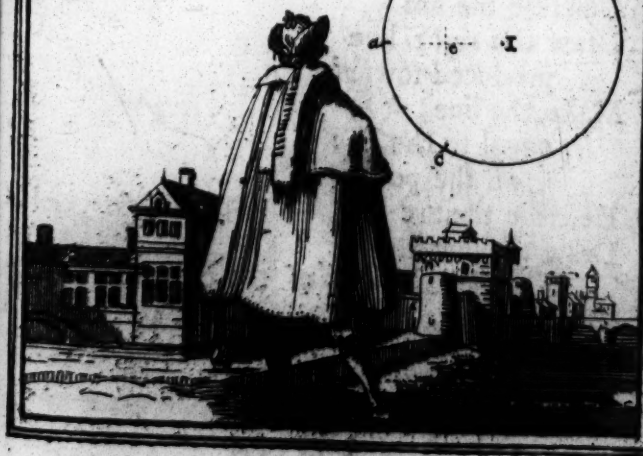
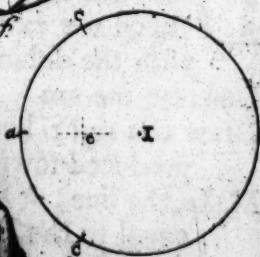
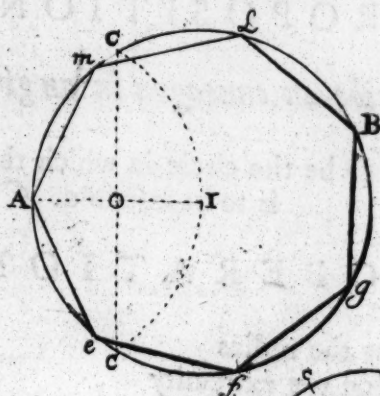
seven times in the circumference of the circle, and the heptagon required will be inscribed.

I





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PROPOSITION V.

To inscribe an enneagon in an given circle

Let B C D be the circle in which the enneagon
is to be inscrib'd.

OPERATION.

Draw the radius
upon the extremity
with the distance

Describe the arc

Draw the right line
produced towards

Make the line
equal to the line
upon the point

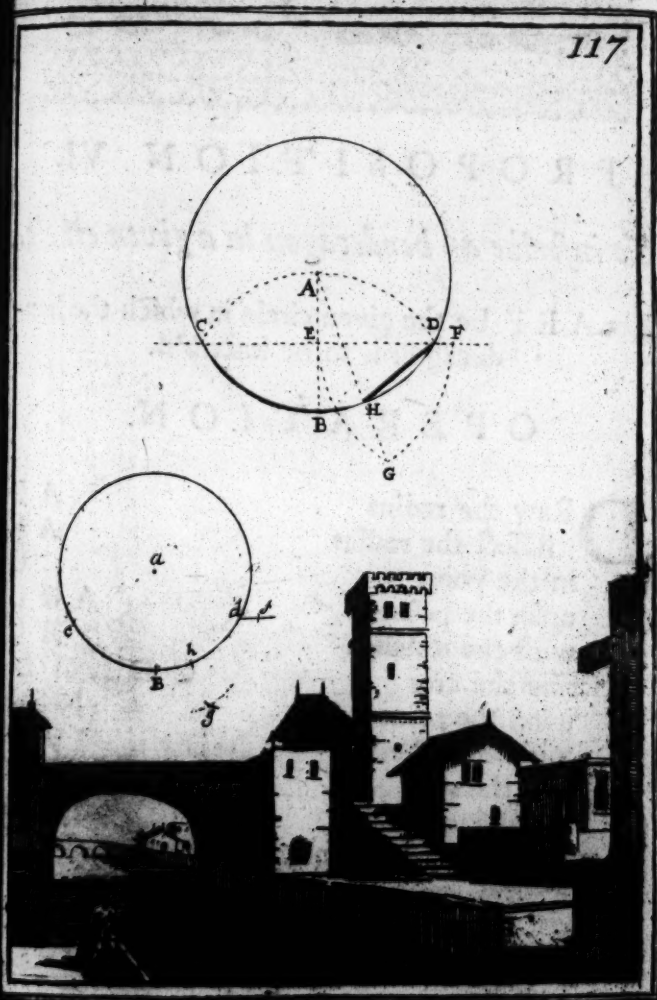
Describe the arc

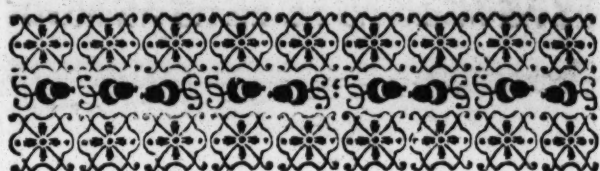
Draw the line

Then the ninth part of the circumference will
be

A B
B A
C A D
C D
E F
A B
E F
E G
A G
D H







PROPOSITION VI.

To inscribe an hendecagon in a given circle.

Let A E F be the given circle in which the hendecagon is to be inscrib'd.

OPERATION.

Pag. 58.

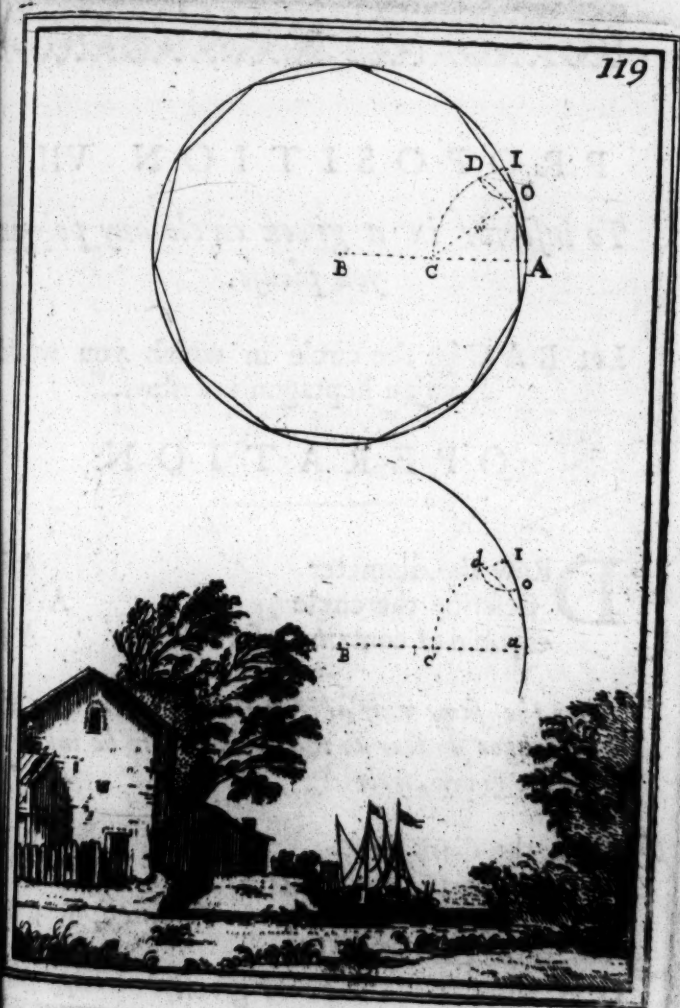
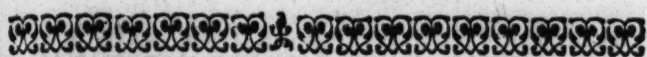
D Raw the radius
Bisect the radius
in the point
upon the points
with the distance

Describe the arcs
upon the point
with the distance

Describe the arc
the distance
will be the side of the hendecagon exact
enough for practice.

A B
A B
C
A & C
A C
C D I, A D.
I
I D
D O
C O







PROPOSITION VII.

*To inscribe in a given circle any polygon
you please.*

Let B A C be the circle in which you would
have an heptagon inscribed.

OPERATION.

Pag. 84.
86. 88.
90.

D Raw the diameter
describe the circle
capable of containing seven times

A B
A B
A B

*After the same way as if you would make upon A B
a polygon similar to that which is to be inscribed
in the given circle*

A B C

Pag. 56. Draw the diameter
parallel to the diameter
Draw the right lines
thro' the extremities

D E
A B
D A G, E B H
D A, E B

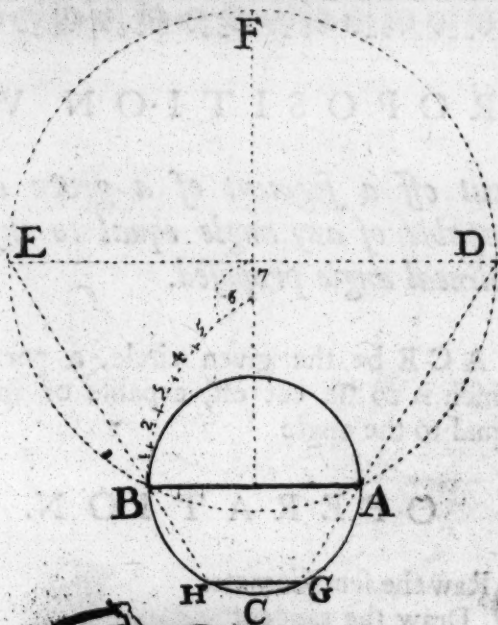
G H will divide the circle given
into seven equal parts.

A B C

After the same Manner act in other polygons.



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P R O P O S I T I O N VIII.

*To cut off a segment of a given circle,
capable of any angle equal to any rec-
tilineal angle proposed.*

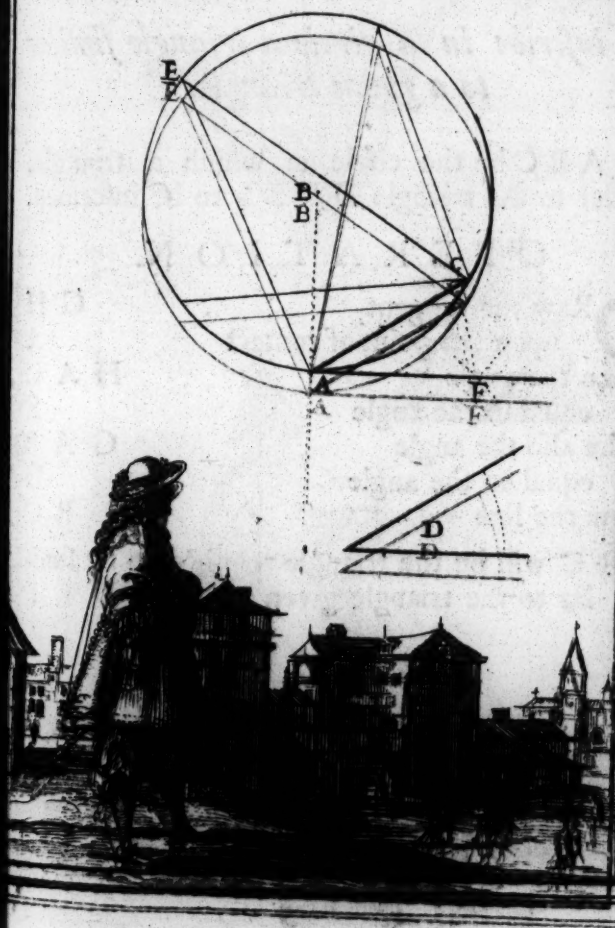
Let A C E be the given circle, a portion of
which is to be cut off, capable of an angle
equal to the angle D

O P E R A T I O N.

Pag. 66. **D**raw the semidiameter
Pag. 62. Draw the tangent
Make the angle
equal to the given angle
All the angles made upon
in the segment
will be equal to the given angle
Therefore the portion
is the segment requir'd.

A B
A F
F A C
D
A C
A E C
D
A E C







PROPOSITION IX.

To inscribe in a circle a triangle similar to a given triangle.

Let A B C be the circle in which a triangle, similar to the triangle D E F is to be inscribed.

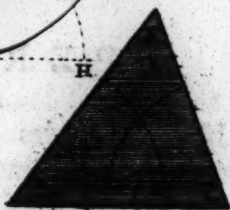
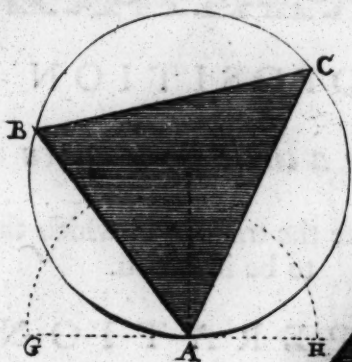
OPERATION.

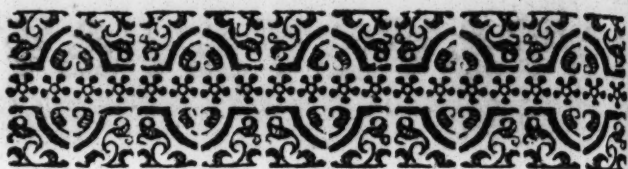
- | | | |
|----------|-------------------------------------------------------------------------|-------|
| Pag. 66. | D raw the tangent | G H |
| | upon the point of contact | A |
| Pag. 62. | Make the angle | H A C |
| | equal to the angle | E |
| Pag. 62. | Make also the angle | G A B |
| | equal to the angle | D |
| | Draw the line | B C |
| | A B C will be the triangle requir'd to be similar to the triangle given | D E F |





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PROPOSITION X.

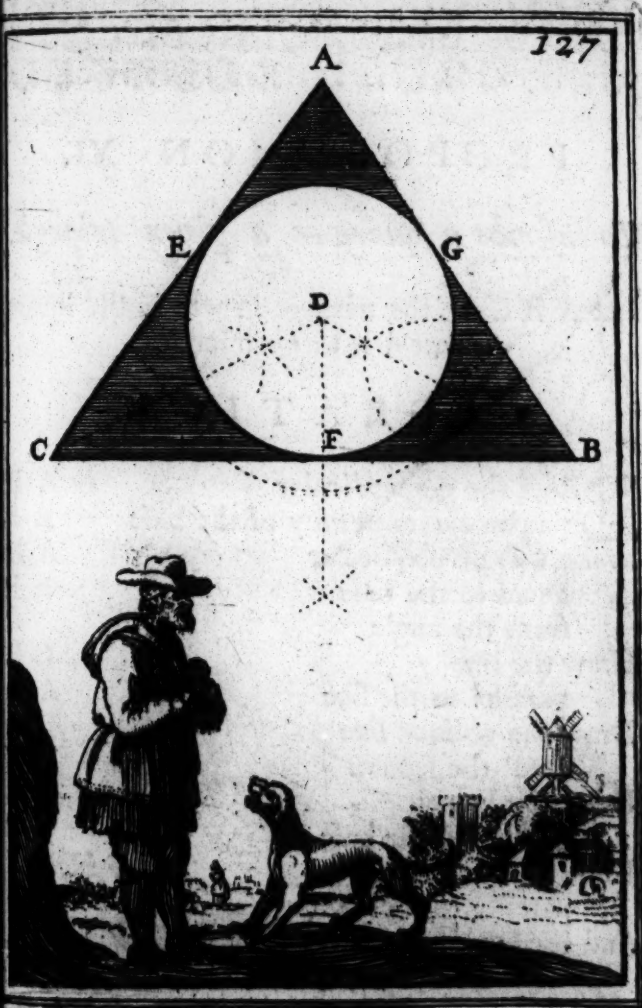
To inscribe a circle in a given triangle.

Let A B C be the triangle in which the circle is to be inscribed.

OPERATION.

Pag. 60.	B Bisect the angles	B & C
	by the right lines	B D & C D
	from the interfection	D
Pag. 54.	Let fall the perpendicular	D F
	Upon the centre	D
	with the distance	D F
	Describe the circle requir'd	E F G







PROPOSITION XI.

To inscribe a square in a given triangle.

Let A B C be the triangle in which the square requir'd is to be inscribed.

OPERATION.

Pag. 50. **E** Rect the perpendicular
upon the extremity of the base
Make this perpendicular
equal to the base
from the angle

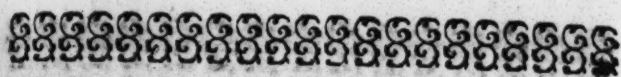
Pag. 56. Draw the line
parallel to the line
Draw the oblique line
thro' the section

Pag. 56. Draw the line
parallel to the base

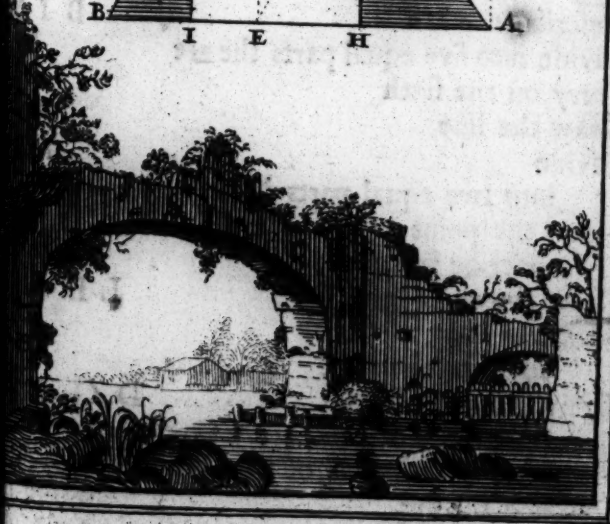
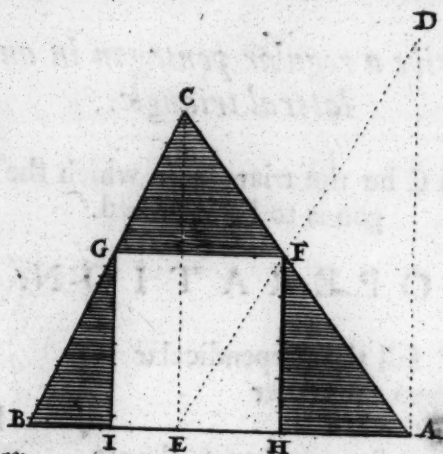
Pag. 56. Draw the lines
parallel to the line
And the square requir'd will be

A D
A B
A D
A B
C
C E
A D
D E
F
F G
A B
F H, G I
C E
F G H I

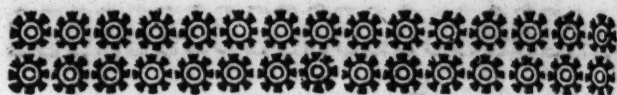




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K



PROPOSITION XII.

To inscribe a regular pentagon in an equilateral triangle.

Let A B C be the triangle in which the pentagon is to be inscribed.

OPERATION.

Pag. 54. **L** E T fall the perpendicular
upon the center

Describe the arc

Divide into five equal parts the arc

Carry on the sixth

Draw the line

Pag. 58. Divide

into two equal parts in

Upon the point

describe the arc

Draw the right line

Make the part

equal to the part

Draw the right lines

upon the center

with the distance of the section

Describe the arc

upon the points

Describe the arcs

Draw the lines

And the pentagon demanded shall be D O P Q N

A I

A

B I M

B I

I M

A M

A M

L

A

L D

L D to H

A G

B H

D G, M C

D

N

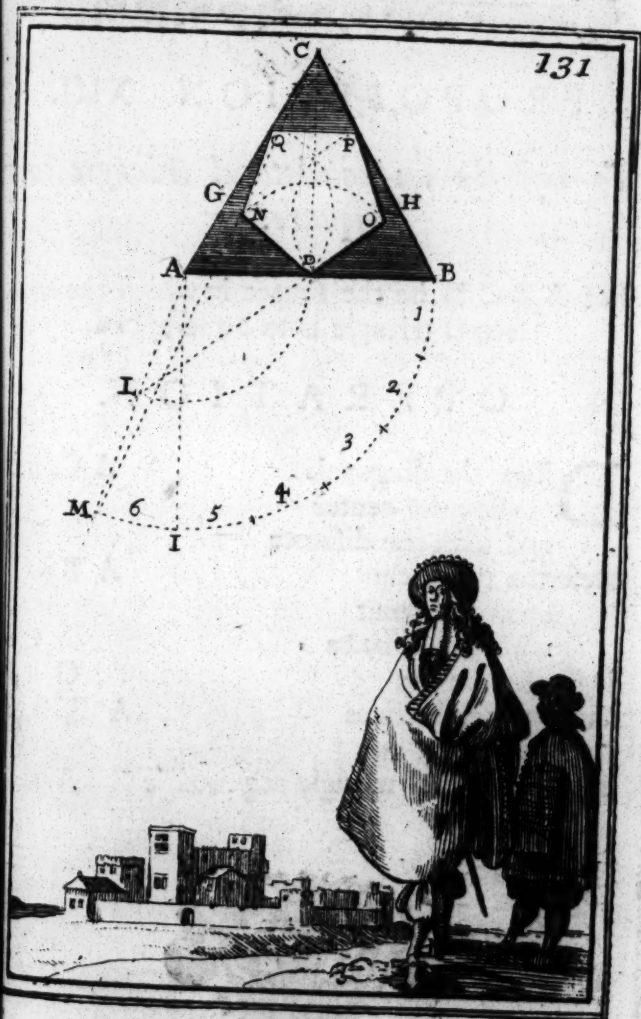
N O

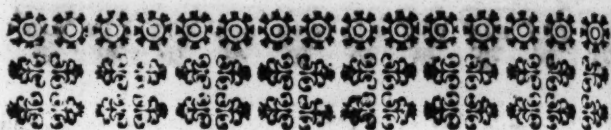
N O

D Q, D P

O P, P Q, N Q

D O P Q N





PROPOSITION XIII.

To inscribe an equilateral triangle in a square.

Let A B C D be the square in which the equilateral triangle is to be inscribed.

OPERATION.

Draw the diagonals
upon the center
and with the distance
Describe the circle
upon the point
with the distance

Describe the arc

Draw the right lines

Draw the right line

The equilateral triangle required is

A C, B D

E

A B C D

C

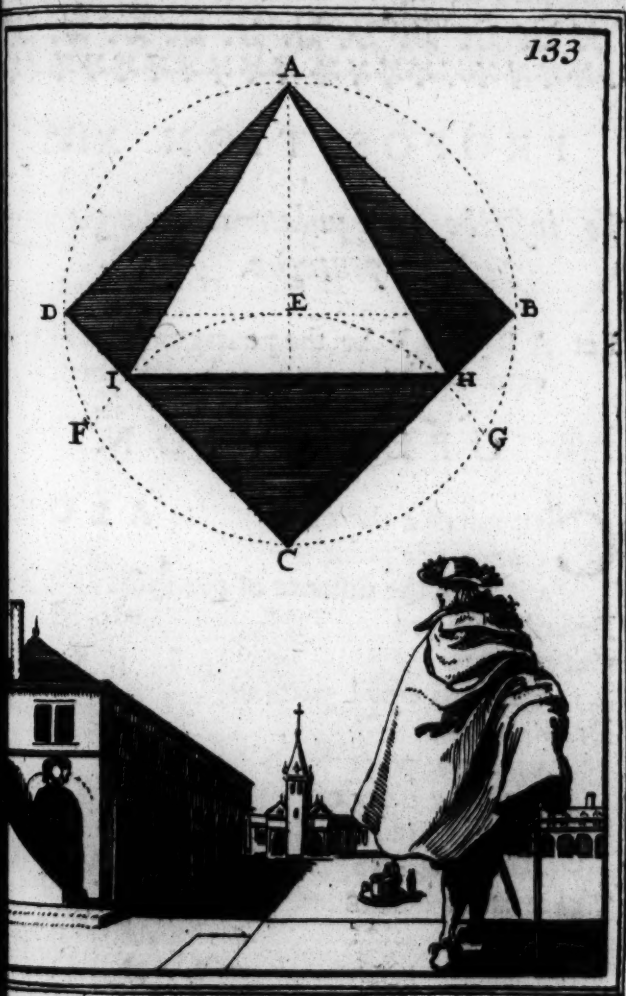
G E

A F, A G

H

A H







PROPOSITION XIV.

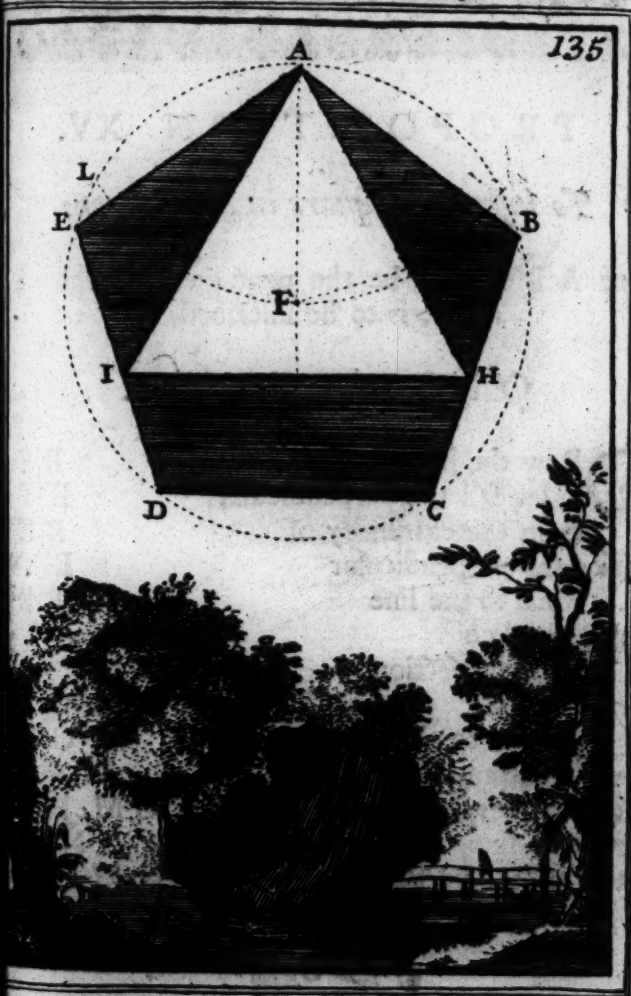
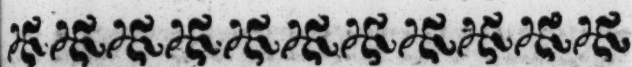
To inscribe an equilateral triangle in a pentagon.

Let A B C D E be the pentagon in which an equilateral triangle is to be inscribed.

OPERATION.

Pag. 98.	C ircumscribe the circle	A B C D
	upon the point	
	and with the distance of the radius	A
	Describe the arc	F
	Cut that arc	F
	into two equal parts in	
	Draw the line	F N
	upon the point	
	with the distance	A
	Describe the arc	H O
	draw the lines	A H, H
	The triangle demanded will be	A H







PROPOSITION XV.

To inscribe a square in a pentagon.

Let A B C D E be the pentagon in which a square is to be inscribed.

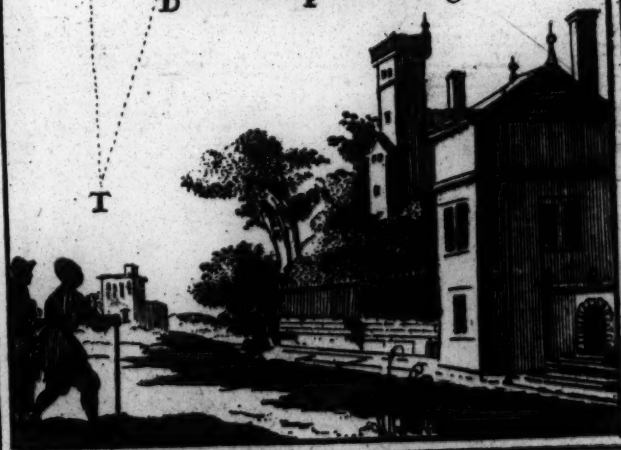
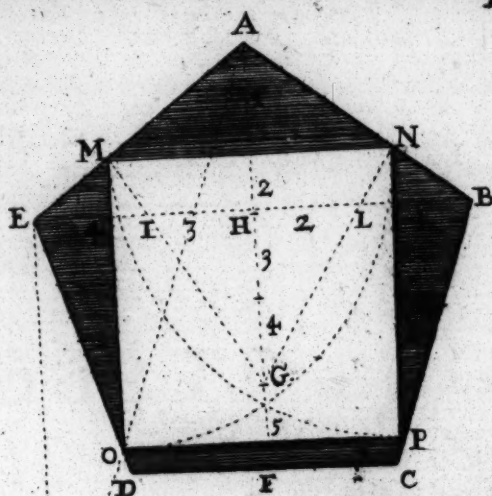
OPERATION.

Pag. 54.	Draw the line	B E
	let fall the perpendicular	E T
	from the extremity of	B E
	Make this perpendicular	E T
	equal to the line	B E
	Draw the line	A T
	thro' the section	O
Pag. 56.	Draw the line	O P
	parallel to the side	C D
	On the extremities	O & P
Pag. 50.	Erect the perpendiculars	O M, P N
	Draw the line	N M
	The square requir'd will be	N M O P





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THE
FOURTH BOOK,

OF THE
Circumscription of Figures.



BOOK the FOURTH.

PROPOSITION I.

To circumscribe a circle about a given triangle.

Let $A B C$ be the triangle about which the circle is to be circumscribed.

OPERATION.

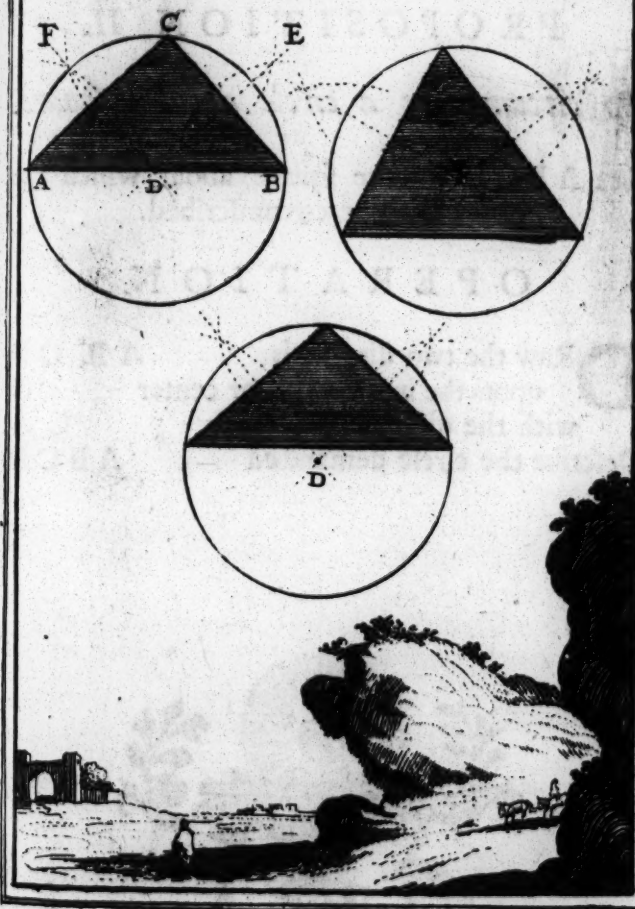
Pag. 98. **D** Escribe the circumference thro' the three points and the thing required will be done.

$A B C$
 A, B, C





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PROPOSITION II.

To circumscribe a circle about a square.

Let ABCD be the square about which the circle is to be circumscribed.

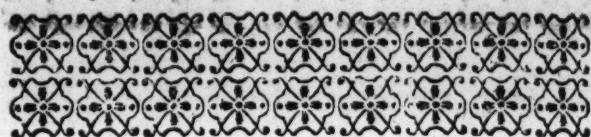
OPERATION.

<p>D Raw the two diagonals upon the intersection or center with the distance Describe the circle demanded</p>	<p>A B, C D G G A A B C D</p>
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OF PRACTICAL GEOMETRY. 143





PROPOSITION III.

To circumscribe a triangle similar to a given triangle, about a given circle.

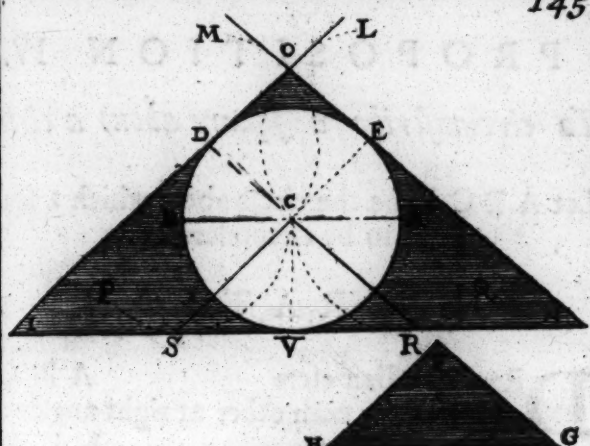
Let $DE V$ be the circle, about which a triangle, similar to the triangle $F G H$, is to be described.

OPERATION.

- | | |
|--------------------------------|------------|
| D Raw the diameter | $A B$ |
| thro' the center | C |
| Pag. 62. Make the angle | $A C E$ |
| equal to the angle | H |
| Pag. 62. Make the angle | $B C D$ |
| equal to the angle | G |
| Produce the lines | $E C, D C$ |
| towards | $R \& S$ |
| Pag. 56. Draw the tangent | $N O$ |
| parallel to the line | $D R$ |
| Pag. 56. Draw the tangent | $O I$ |
| parallel to the line | $E S$ |
| Pag. 56. Draw also the tangent | $N I$ |
| parallel to the diameter | $A B$ |
- $I N O$ will be the triangle requir'd similar to the triangle $F G H$, and circumscribed about the circle $DE V$.



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P R O P O S I T I O N I V .

To circumscribe a square about a circle

Let A B C D be the circle about which a square
is to be circumscribed.

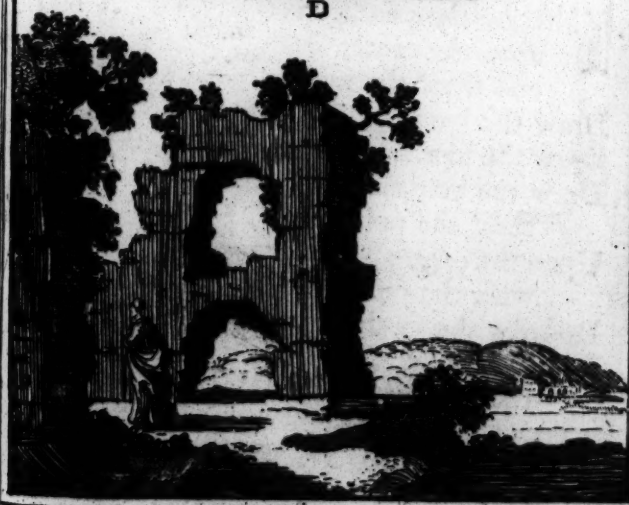
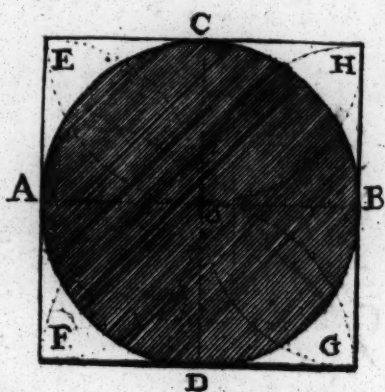
O P E R A T I O N .

D Raw the diameters	A B, C
intersecting each other at right angles in	
upon the points	A, C, B,
with the distance	A
Describe the semicircles	H O G, H O
	E O F, F O
Draw the right lines	E F, F G, G H, H
thro' the intersections	E, F, G,
The square demanded will be	E F G





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PROPOSITION V.

To circumscribe a pentagon about a given circle.

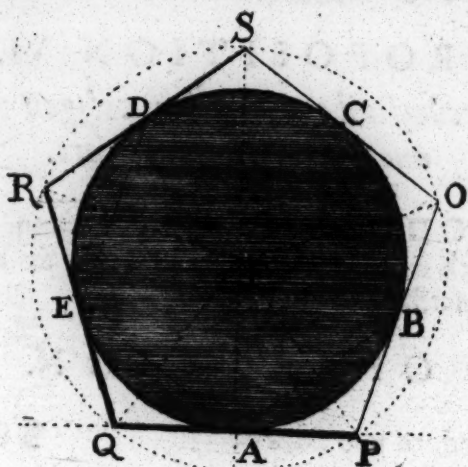
Let A B C D E be the given circle about which a pentagon is to be circumscribed.

OPERATION.

- Pag. 411. **I**nscribe the pentagon upon the center and thro' the middle of each side
 Draw the lines F O, F P, F Q, F R, F S
 Draw the line F A
 Pag. 68. Draw the tangent thro' the point P
 Upon the centre F
 with the radius F R
 Describe the circle O P Q R
 Draw the Sides of the pentagon demanded thro' the sections O, P, Q, R,



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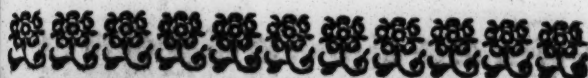
PROPOSITION VI.

To circumscribe a regular polygon about another of the same sort.

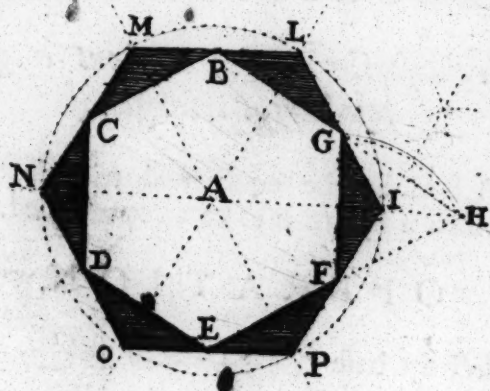
Let B C D E F G be the polygon given, about which another similar polygon is to be circumscribed.

OPERATION.

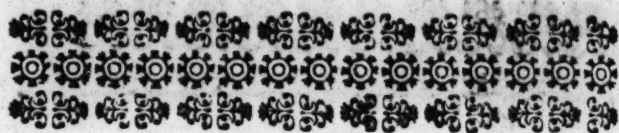
<p>Produce two sides as until they meet in Draw the line Pag. 60. Draw the line bisecting the angle upon the centre with the distance Describe the arc Draw the radius's thro' the middle of each side. Draw the sides of the exterior polygon demand- ed, thro' the sections</p>	<p>B G, E F H A H E I G F H A A I I M O A L, A M, A N, A O I, L, M, N, O, P</p>
-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------



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L4



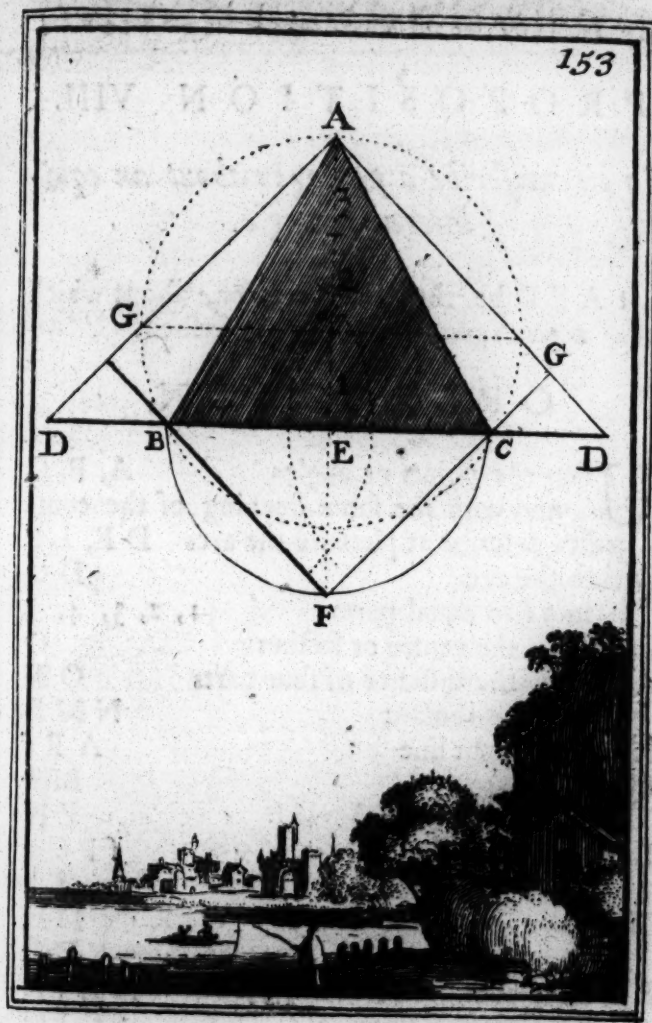
PROPOSITION VII.

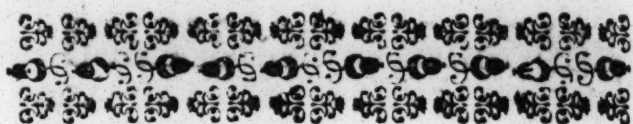
To circumscribe a square about a given equilateral triangle.

Let A B C be the equilateral triangle, about which a square is to be circumscribed.

OPERATION.

Pag. 58. B Isect the base	BC
in the point	E
Produce the base	BC
both ways towards	D & D
Make the lines	ED & ED
equal to the line	EA
Upon the point	E
with the distance	EC
Describe the semicircle	BFC
draw the line	A EF
From the point	F
draw the lines	FCG & FBG
and the square requir'd will be	AGFG





P R O P O S I T I O N VIII.

To circumscribe a pentagon about an equilateral triangle.

Let A B C be the triangle given, about which
a pentagon is to be circumscribed.

O P E R A T I O N.

U	Pon the points or angles	A, B, C
	and with the same opening of the compasses describe at pleasure the arcs	D E, L P
	Divide the arc	D O
	into five equal parts	1, 2, 3, 4, 5
	upon the centre or section	O
	And with the distance of four parts	O N
	describe the arc	N M E
	Draw the right line	A E f
	Cut off the arc	M P
	equal to the arc	E N
	Draw the right line	f P C g
	equal to the line	f A
	Make the arc	D H
	equal to the arc	D E
	Draw the sides	A I, I R
	equal to the sides	A f, f G
	The side	I R
	will compleat the pentagon demanded.	



PROPOSITION IX.

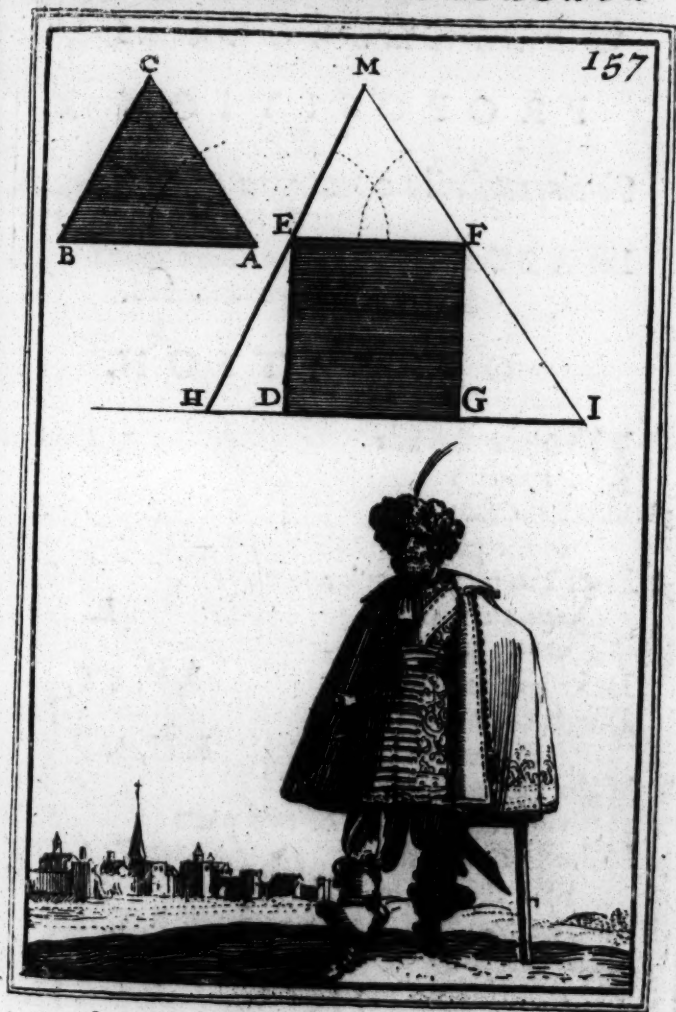
To circumscribe a triangle similar to a given triangle, about a square.

Let D E F G be the square about which a triangle is to be circumscribed similar to the triangle A B C.

OPERATION.

Pag. 62.	M Ake the angle	E F M
	equal to the angle	A
Pag. 62.	Make the angle	M E F
	equal to the angle	B
	Produce the lines	M E, M F, D G
	towards	I & H
	H I M will be the triangle requir'd, similar to	A B C
	the triangle	
	and circumscrib'd about the square D E F G	







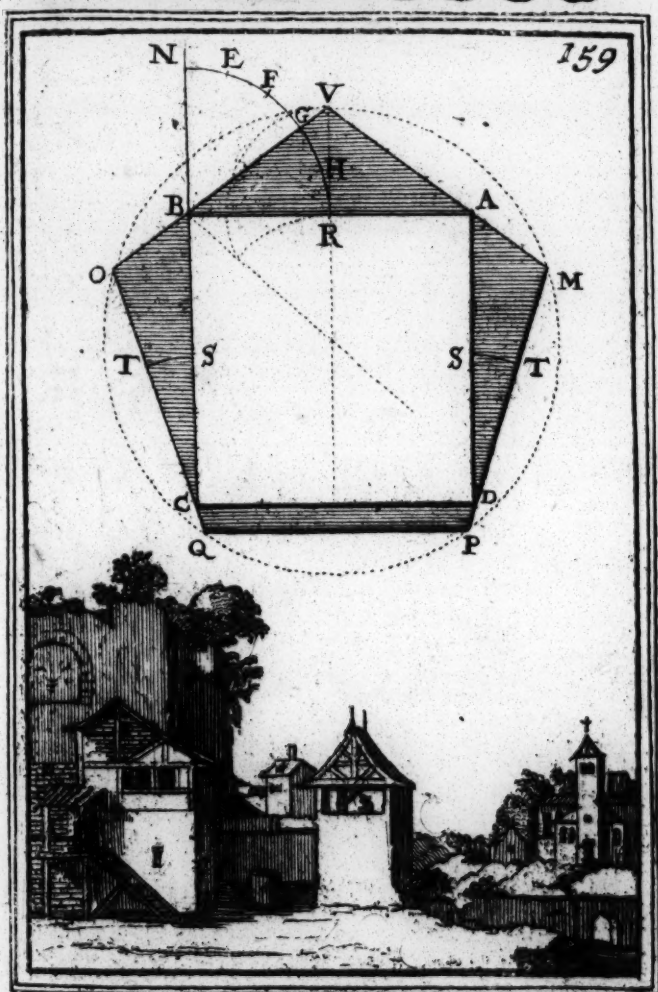
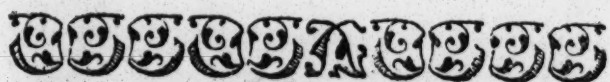
PROPOSITION X.

To circumscribe a pentagon about a square.

Let A B C D be the square about which a pentagon is to be circumscribed.

OPERATION.

Produce the side	CB
towards	N
Pag. 58. Bisect the side	AB
in the point	R
Pag. 56. Erect the perpendicular	RV
upon the points	B, D, C
with the distance	BR
Describe the arcs	RN, ST, ST
Divide the arc	RN
into five equal parts	RH, GF, EN
Make the angle	RBV
with the distance of two parts	RG
Make the angles	SCT, SDT
with the distance of one part	RH
Produce the lines	VB, CT to O
Make the line	OQ
equal to the line	OV
Draw the other sides after the same manner, and	
you will have the Thing requir'd.	



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THE
FIFTH BOOK
OF
Proportional LINES.

M



BOOK the FIFTH.

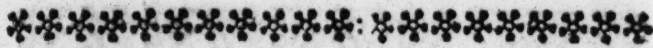
PROPOSITION I.

To find a mean proportional between two given lines.

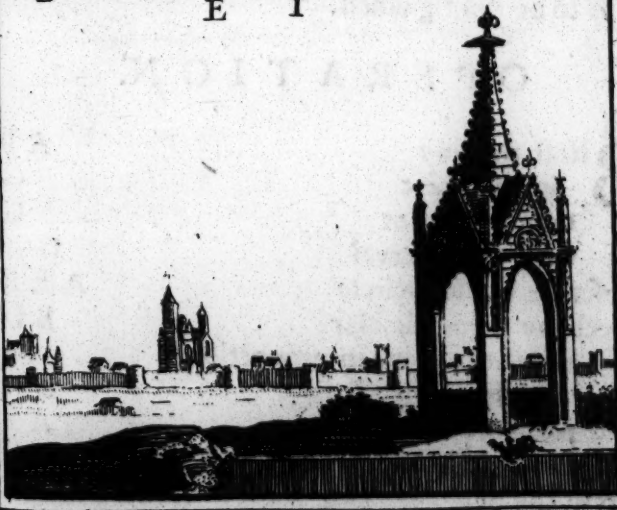
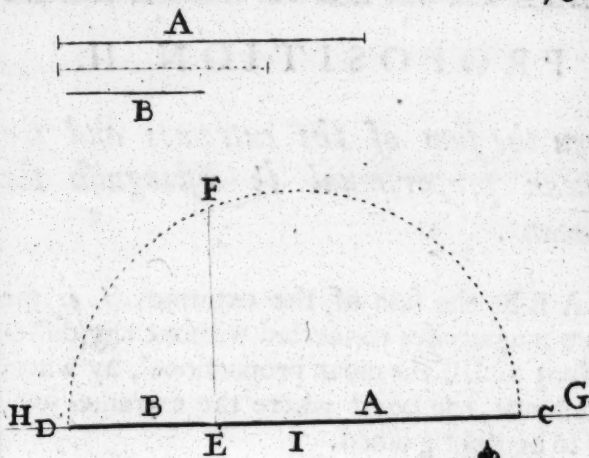
Let A & B be the two lines between which a mean proportional is to be found.

OPERATION.

D Raw an indetermin'd Line	GH
Make	CE
equal to the line	A
Make	ED
equal to the line	B
Pag. 58. Bisect	CD
in the point	F
upon the point	I
and with the distance	IC
Describe the semicircle	CFD
Erect the perpendicular	EF
This line	EF
shall be a mean proportional between A & B	



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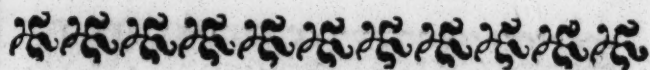
PROPOSITION II.

Given the sum of the extremes and the mean proportional to distinguish the means.

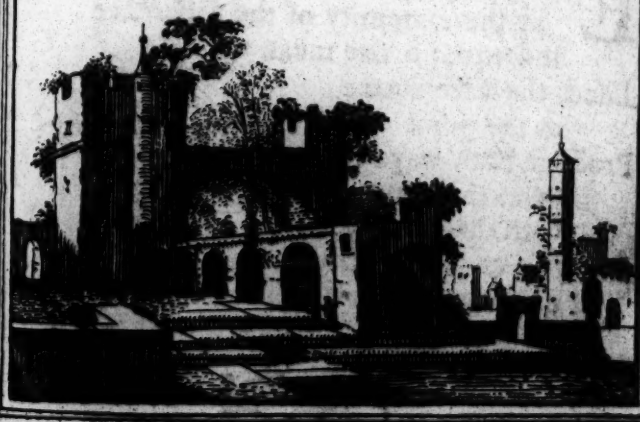
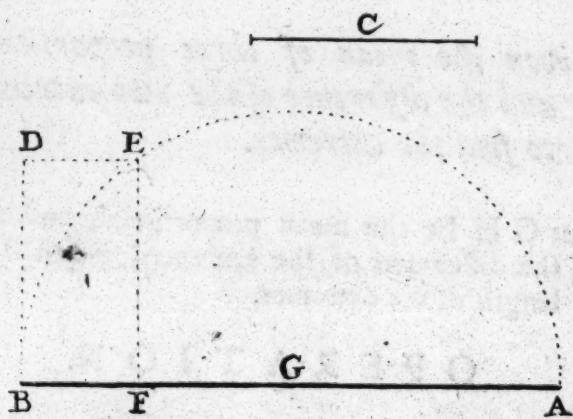
Let A B be the sum of the extremes (*i. e.* the two magnitudes connected without any distinction) and C the mean proportional, by whose assistance the point where the extremes join, is to be distinguished.

OPERATION.

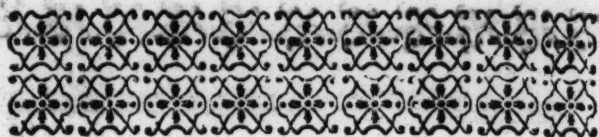
Pag. 53.	B Isect the line	A B
	in the point	G
	upon the point	G
	with the interval	G A
	Describe the semicircle	A E B
	Erect the perpendicular	B D
	equal to the mean proportional	C
Pag. 56.	Draw the line	D E
	parallel to the line	A B
	from the section	E
Pag. 56.	Draw the line	E F
	parallel to the line	B D
	Then will the point where the extremes join be	F
	so that C or its equal	E F
	shall be a mean proportional	
	between	A F & B F



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M 3



PROPOSITION III.

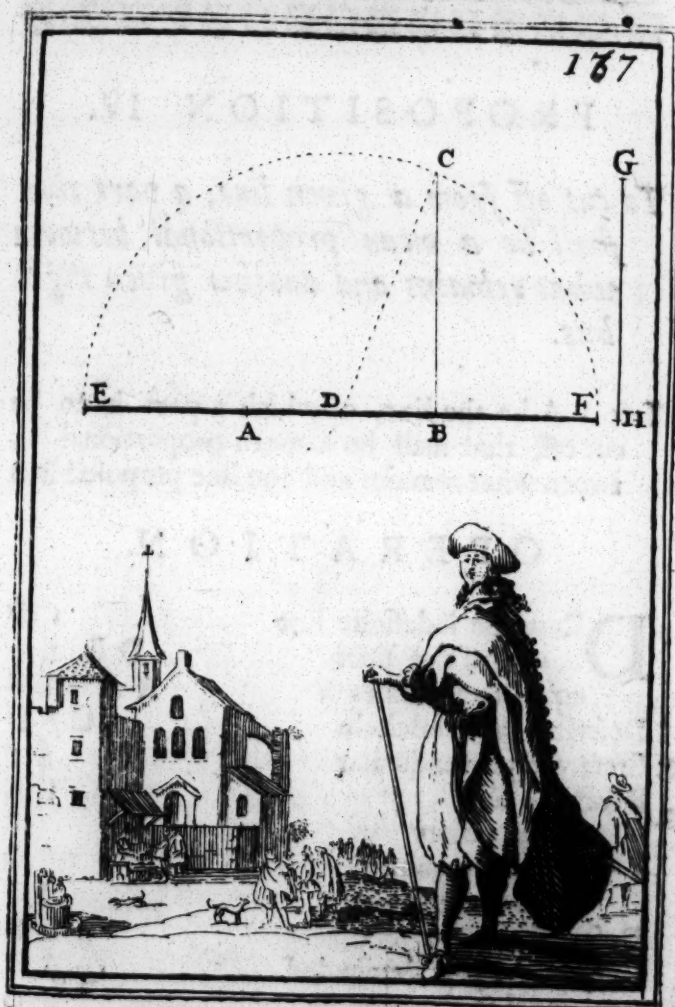
*Given the mean of three proportionals
and the difference of the two extremes,
to find the extremes.*

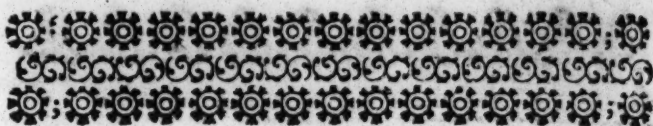
Let G H be the mean proportional, and A B
the difference of the extremes, requir'd the
length of the extremes.

OPERATION.

Pag. 50.	E Rect the perpendicular	B C
	at the extremity of the difference	A B
Pag. 58.	and equal o the mean	G H
	Bisect the difference	A B
	in the point	D
	Produce both ways towards	E & F
	upon the point	D
	with the distance	D C
	Describe the semicircle	E C F
	The extremes requir'd will be	B E, B F







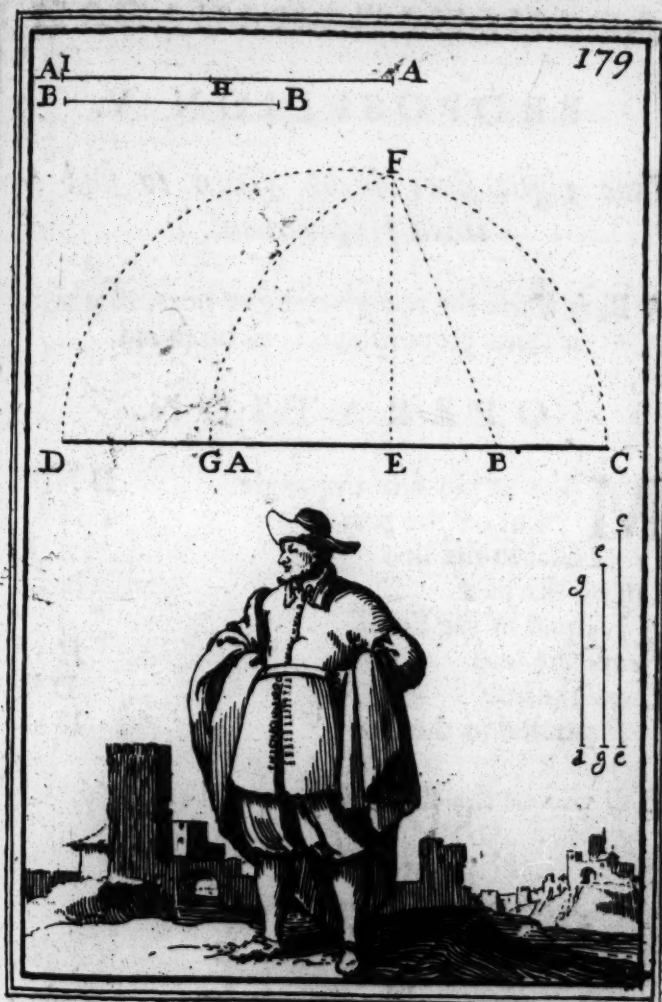
PROPOSITION IV.

To cut off from a given line, a part that shall be a mean proportional between what remains and another given right line.

Let A A be the line, of which a part is to be cut off, that shall be a mean proportional between what remains and the line proposed B B.

OPERATION.

D Raw the indefinite line	CD
cut off the lines	DE, EC
equal to the lines	AA & BB
Describe the semicircle	CFD
Pag. 46. Erect the perpendicular	EF
Pag. 58. Bisect the line	CE
in the point	B
upon the point	B
with the distance	BF
Describe the arc	FG
Cut off the part demanded	AH
equal to the part	EG
A H will be the mean proportional between the remainder	HI
and the other line proposed	BE





PROPOSITION V.

Two right lines being given to find a third proportional.

A B, A C are the two given right lines, to which a third proportional is to be found.

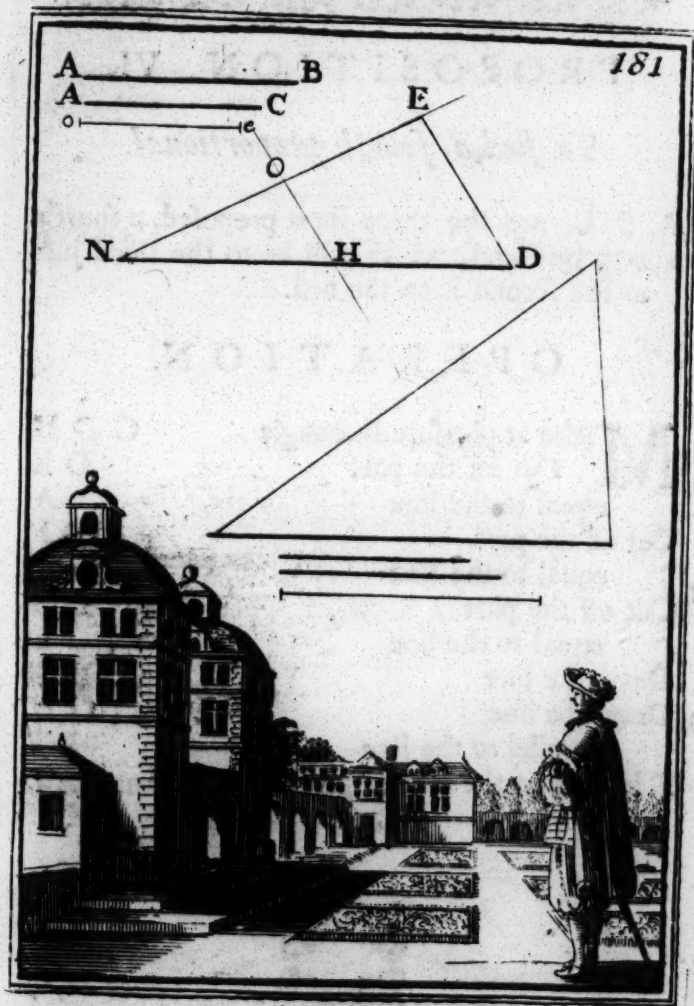
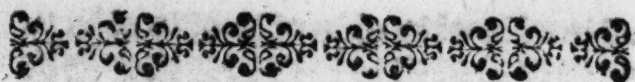
OPERATION.

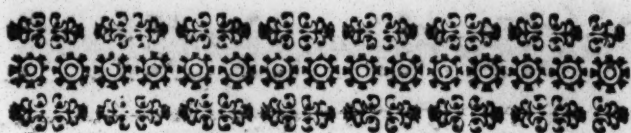
M ake at pleasure the angle	D N E
Cut off the part	N H
equal to the line	A B
Cut off the part	N O
equal to the line	A C
Draw the line	H O
Draw the line	D E
parallel to the line	H O

Fig. 56.

E O will be the third proportional requir'd.







PROPOSITION VI.

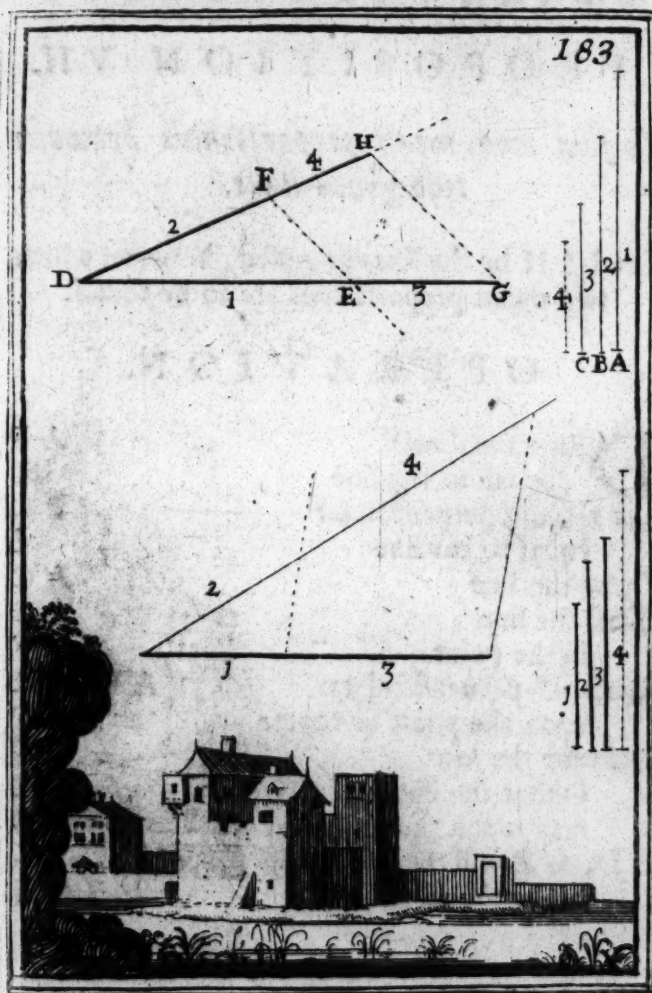
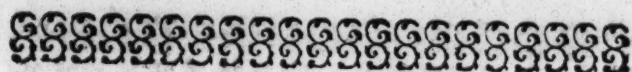
To find a fourth proportional.

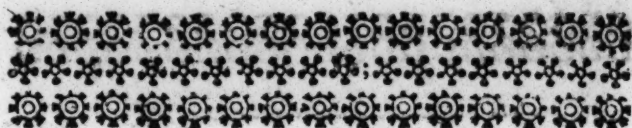
A, B, C, are the three lines proposed, a fourth is to be found, which will be to the third just as the second is to the first.

OPERATION.

M ake at pleasure the angle	G D H
Cut off the part	D E
equal to the line	A
Cut off the part	D F
equal to the line	B
Cut off the part	E G
equal to the line	C
Draw the line	E F
Fig. 56. Draw the line	G H
parallel to the line	E F
F H will be the fourth proportional demanded.	







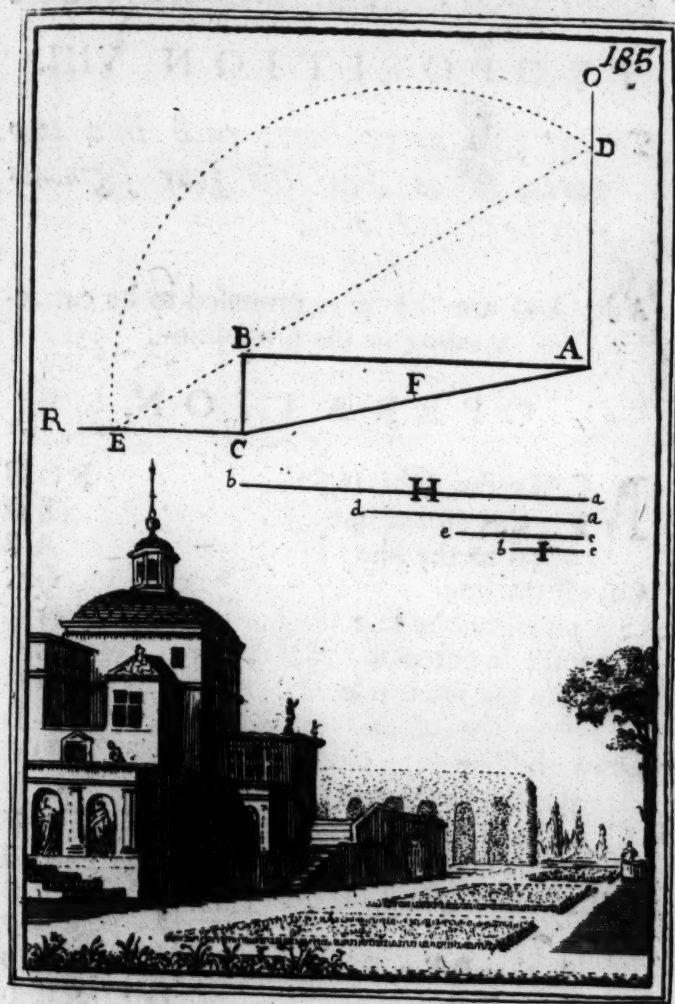
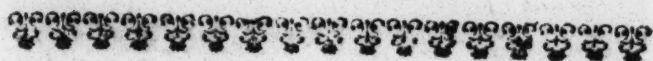
P R O P O S I T I O N VII.

To find two mean proportionals between two given lines.

Let I & H be the lines proposed, between which two mean proportionals are to be found.

O P E R A T I O N.

D Raw the line	A B
equal to the line	H
Let fall the perpendicular	B C
equal to the line	I
Draw the line	A C
Pag. 58. Bisect the line	A C
in the point	F
Pag. 50. Erect the perpendiculars	A O, C R
upon the point or center	F
Describe the arc	D E
so that the chord	D E
may touch the angle	B
A D, C E will be the mean proportionals be-	
tween the given lines	I & H





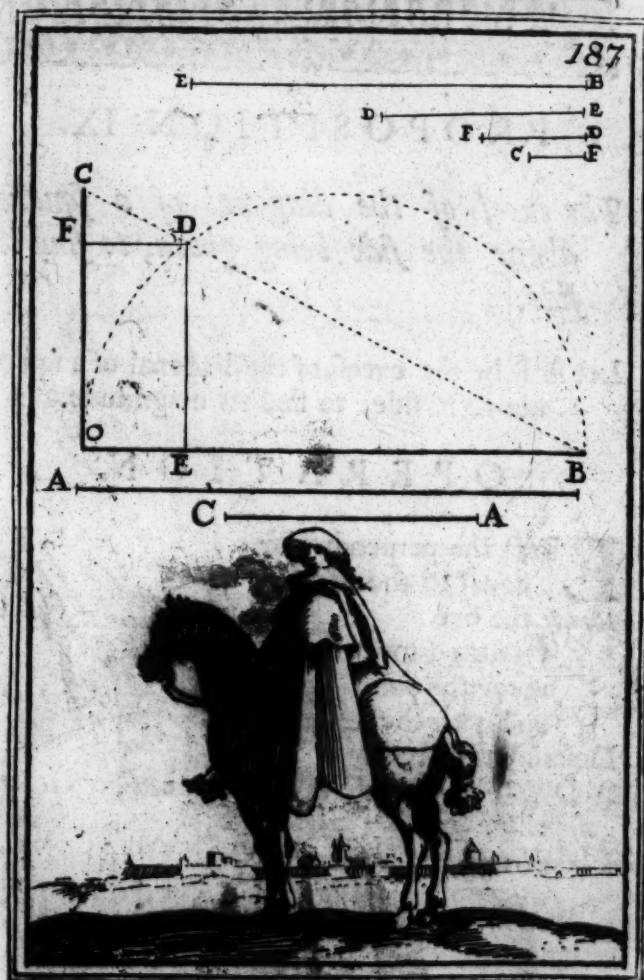
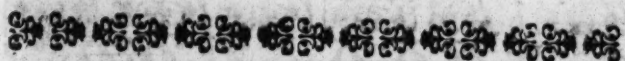
PROPOSITION VIII.

To cut two given lines, each into two parts, so as that the four segments may be proportional.

A B, A C are the lines proposed to be cut according to the proposition.

OPERATION.

M ake the right angle	B O C
Cut off the line	B O
equal to the line	A B
Cut off the line	O C
equal to the line	A C
Draw the hypotenuse	B C
Describe the semicircle	B D O
from the section	D
Pag. 56. Draw the line	D E
parallel to the line	C O
Pag. 56. and the line	D F
parallel to the line	E O
A B will be cut in	E
O C also in	F
so that B E will be to	E D
as E D to	D F, & E D
to D F, as D F is to	F C





PROPOSITION IX.

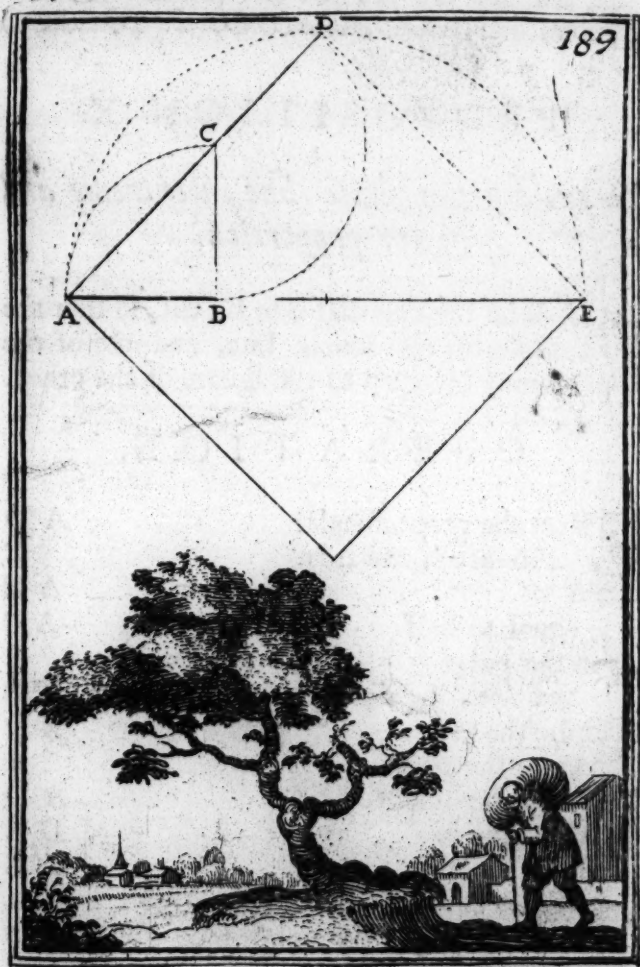
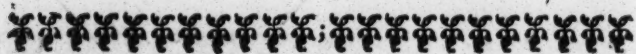
*The excess of the diagonal of a square,
above the side being given, to find its
side.*

Let A B be the excess of the diagonal of a square
above its side, to find its magnitude.

OPERATION.

Pag. 50.	E Rect the perpendicular	B C
	equal to the excess	A B
	Draw the line	A C
	produced towards	D
	upon the point	C
	and with the distance	C B
	Describe the arc	B D
	A D will be the side of the square	A
	the excess	A B
	of whose diagonal	A E
	above the said side	A D







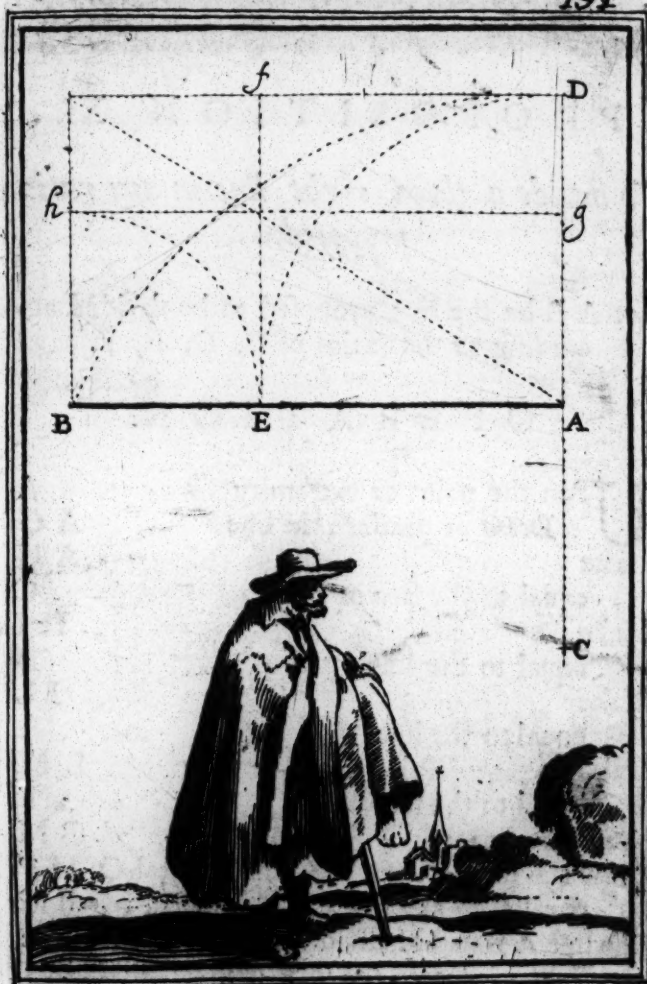
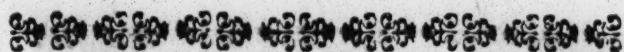
PROPOSITION X.

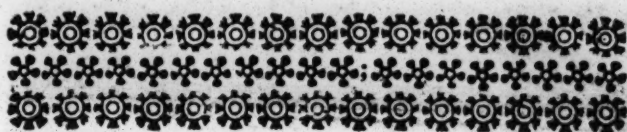
To cut a given finite line in extreme and mean proportion.

Let A B be the line that is to be cut, so that the rectangle of the whole line, and one of the parts may be equal to the square of the other.

OPERATION.

Pag. 50.	E Rect the perpendicular	A D
	Produce it towards	C
	Make	A C
	equal to half	A B
	Upon the point	C
	and with the distance	C B
	Describe the arc	B D
	upon the point	A
	with the distance	A D
	Describe the arc	D E
	The line	A B
	will be cut in the point	A
	in the proportion requir'd : for if you make	
	the rectangle A h of the whole A B and	
	part B E, it will be equal to the square	
	A f made upon the other part	A E





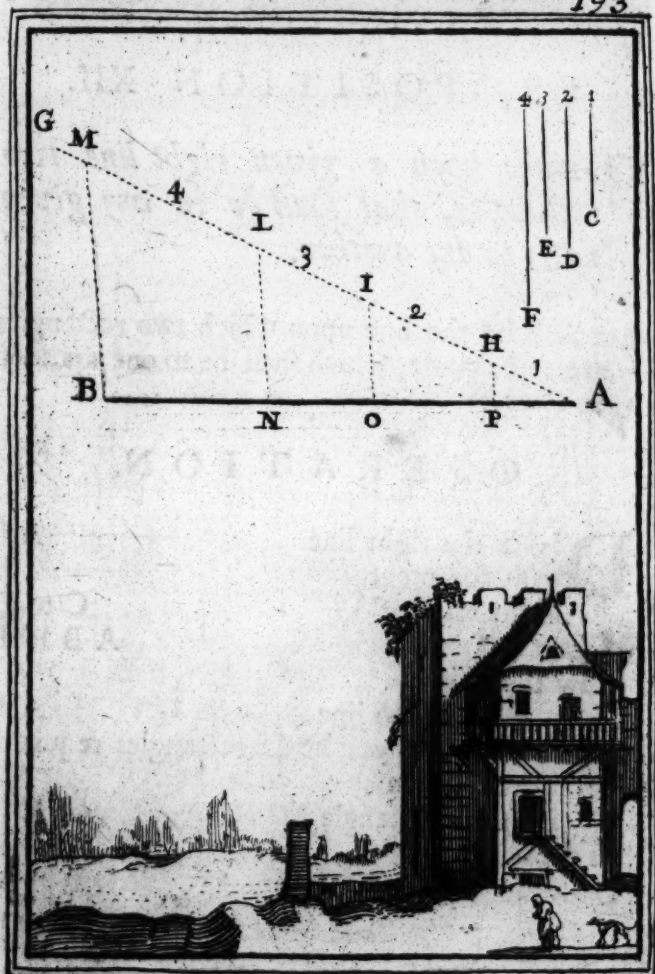
P R O P O S I T I O N X I.

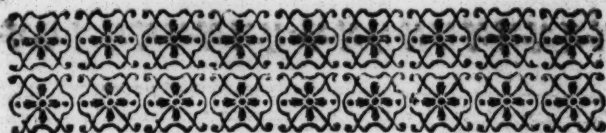
*To divide a given right line in any ratio
proposed.*

Let A B be the line proposed to be divided ac-
cording to the ratios of C, D, E, F.

O P E R A T I O N.

U pon the point or extremity	A
Draw at pleasure the line	A G
Make	A H
equal to the line or ratio	C
Make	H I
equal to the line	D
Make	I L
equal to the line	E
Make	L M
equal to the line	F
Draw the line	B M
Fig. 50. Draw the lines	L N, I O, H P
parallel to the line	B M
The line A B will be divided in the points P, O, N	
according to the ratio demanded.	





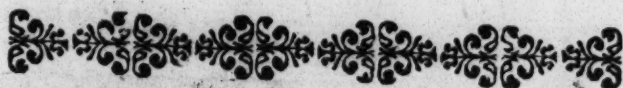
PROPOSITION XII.

To make upon a given right line two rectangles, that shall be in any given ratio to one another.

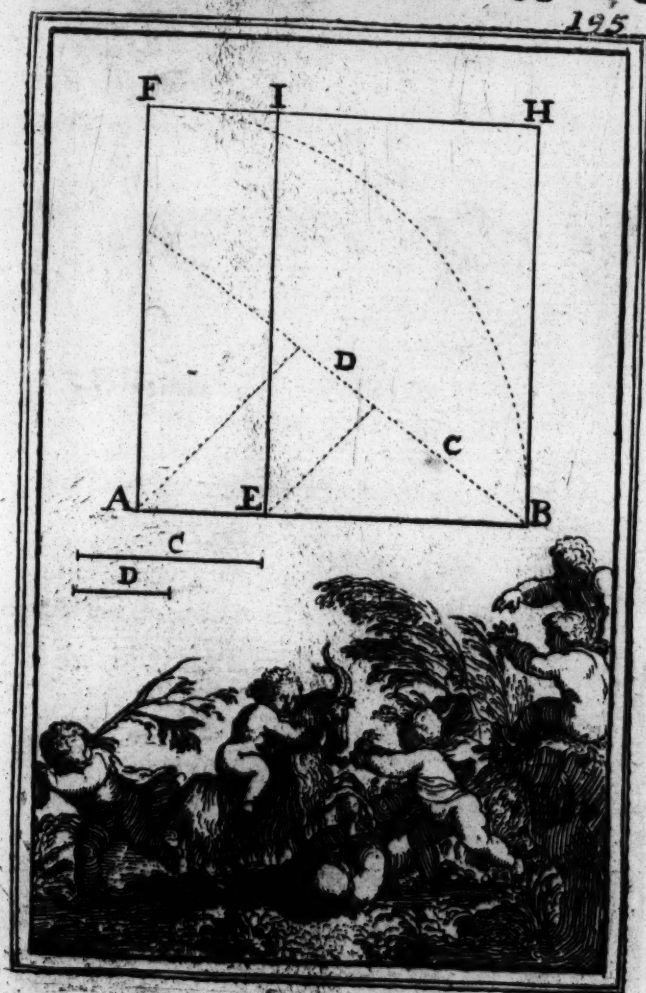
Let A B be the line upon which two rectangles are to be made, which shall be to one another as C to D

OPERATION.

Pag. 184	D Ivide the right line	A B
	at the point	E
	in the ratio of	C to D
Pag. 82.	Make the square	A B H F
Pag. 56.	Draw the line	E I
	parallel to the line	A F
	B E I H, A E I F will be the rectangles requir'd.	A I
	the rectangle	E H
	is to the rectangle	D
	As the line	C
	is to the line	



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BENE, A. H. H.
J. H. H. H. H. H. H.
E. H. H. H. H. H. H.
J. H. H. H. H. H. H.
S. E. L. E. R. O. T. J. H. H.
J. N. G. G. O. H. H.
D. I. H. H. H. H. H. H.
O. N. S. H. H. H. H.
H. N. E.

